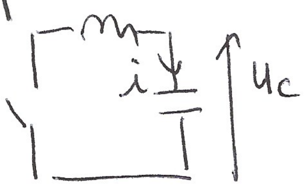
Correction TD circuits 2nd ordre

en régime transitoire

Ex1



1) à $t=0^-$ $i=0$ (K ouvert) $\Rightarrow$ $i(0^+)=0$

$u=u_0$ (C chargé) $\Rightarrow$ $u(0^+)=u_0$.

car i traversant la bobine et u tension aux bornes de C sont continus.

2) $\begin{cases} L\dfrac{di}{dt}+u_C=0 \\ i=C\dfrac{du_C}{dt} \end{cases}$ $\rightarrow$ $LC\dfrac{d^2u_C}{dt^2}+u_C=0$ $\quad \dfrac{d^2u_C}{dt^2}+\dfrac{1}{LC}u_C=0$

à l'identifier à $\left.\dfrac{d^2u_C}{dt^2}+\omega_0^2u_C=0\right\}\Rightarrow \omega_0^2=\dfrac{1}{LC}$

$u_C(t) = A\cos(\omega_0 t+\varphi)$ et $u_C(0^+)=u_0 = A\cos\varphi$

$i(0^+)=C\dfrac{du_C}{dt}(0^+)=0=-A\omega_0\sin\varphi \Rightarrow \varphi=0.$

d'où $\boxed{u_C(t)=u_0\cos\omega_0 t}$

3) $i=C\dfrac{du_C}{dt}=-C\omega_0 u_0\sin\omega_0 t=-C\cdot\dfrac{u_0}{\sqrt{LC}}\sin\omega_0 t=-\sqrt{\dfrac{C}{L}}\cdot u_0\sin\omega_0 t$

$\boxed{i(t)=\sqrt{\dfrac{C}{L}}u_0\cos\left(\omega_0 t+\dfrac{\pi}{2}\right)}$ Rq $\left[\sqrt{\dfrac{C}{L}}\right]=\sqrt{\dfrac{[C]}{[L]}}=\sqrt{\dfrac{T/[R]}{T\cdot[R]}}$ ..

$=\dfrac{1}{[R]}$

donc $\left[\sqrt{\dfrac{C}{L}}\cdot u_0\right]=\dfrac{[\text{Tension}]}{[\text{Resistance}]}$

$u_L(t)=L\dfrac{di}{dt}=-L\cdot\sqrt{\dfrac{C}{L}}\cdot u_0\cdot\dfrac{1}{\sqrt{LC}}\cos\omega_0 t=\boxed{-u_0\cos\omega_0 t=-u_C(t)}$ (prévisible car $u_C+u_L=0$.)

4) $(u_L+u_C)\cdot i=0$

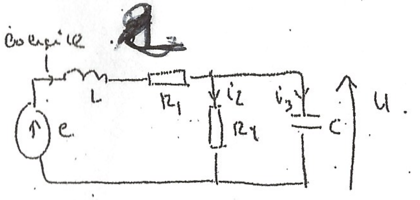
$P_C+P_L=0$

$\dfrac{d}{dt}(\mathcal{E}_C+\mathcal{E}_L)=0 \Rightarrow \mathcal{E}_C+\mathcal{E}_L=\dfrac{1}{2}Cu_C^2(t)+\dfrac{1}{2}Li^2(t)=\text{cste}=\mathcal{E}_C(t=0)+\mathcal{E}_L(t=0)$

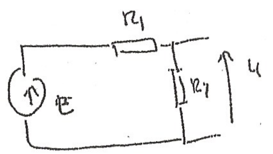
$=\dfrac{1}{2}Cu_0^2$

$$\boxed{\dfrac{1}{2}Cu_C^2(t)+\dfrac{1}{2}Li^2(t)=\dfrac{1}{2}Cu_0^2}$$

2nd ordre régime transitoire



lorsque $t \to \infty$: C = circuit ouvert ; L = fil



$$\boxed{u(\infty) = \frac{R_2}{R_1+R_2} \cdot E}$$

(diviseur de tension).

$$e = L\frac{di}{dt} + R_1 i + u \quad (1)$$

$$\left.\begin{array}{l} i = i_2 + i_3 \\ i_3 = C\frac{du}{dt} \\ i_2 = \frac{u}{R_2} \end{array}\right\} \Rightarrow i = C\frac{du}{dt} + \frac{u}{R_2}$$

d'où (1) $\Rightarrow$ $e = L\left[C\frac{d^2u}{dt^2} + \frac{1}{R_2}\frac{du}{dt}\right] + R_1 C\frac{du}{dt} + \frac{R_1}{R_2}u + u.$

$$\Rightarrow \boxed{LC\frac{d^2u}{dt^2} + \left(\frac{L}{R_2} + R_1 C\right)\frac{du}{dt} + u\left(\frac{R_1}{R_2}+1\right) = e.}$$

$$\frac{d^2u}{dt^2} + \frac{\left(\frac{L}{R_2} + R_1 C\right)}{LC}\cdot\frac{du}{dt} + \frac{u}{LC}\left(\frac{R_1}{R_2}+1\right) = \frac{e}{LC}$$

$$\left\{\begin{array}{l} \omega_0^2 = \frac{1}{LC}\left(\frac{R_1}{R_2}+1\right) \Rightarrow \omega_0 = \sqrt{\frac{R_1+R_2}{R_2 \cdot LC}}. \\ 2\lambda\omega_0 = \frac{\frac{L}{R_2} + R_1 C}{LC} \Rightarrow \lambda = \frac{1}{2}\frac{\left(\frac{L}{R_2}+R_1 C\right)}{LC}\sqrt{\frac{R_2}{R_1+R_2}}\cdot\sqrt{LC} \Rightarrow \boxed{\lambda = \frac{1}{2}\frac{\left(\frac{L}{R_2}+R_1 C\right)}{\sqrt{LC}}\sqrt{\frac{R_2}{R_1+R_2}}} \\ \quad = \frac{1}{R_2 C} + \frac{R_1}{L} \end{array}\right.$$

$$\boxed{u(t) = Ae^{-\lambda\omega_0}\cos\left(\omega_0\sqrt{1-\lambda^2}\,t + \varphi\right) + E\cdot\frac{R_2}{R_1+R_2}.}$$

(1)

$$\delta = \ln\frac{u(t)-u(\infty)}{u(t+T)-u(\infty)} = \ln\frac{5-2}{8{,}5-2} = \boxed{1{,}4 = \delta} = \ln e^{+\lambda\omega_0 T}$$

$$\delta = \lambda\,\omega_0\cdot\frac{2\pi}{\omega_0\sqrt{1-\lambda^2}} \Rightarrow \boxed{\delta = \frac{2\pi\lambda}{\sqrt{1-\lambda^2}}} \Rightarrow \delta^2 = \frac{4\pi^2\lambda^2}{1-\lambda^2}$$

$$\delta^2(1-\lambda^2) = 4\pi^2\lambda^2 \Rightarrow \lambda^2\left[4\pi^2 + \delta^2\right] = \delta^2 \Rightarrow \boxed{\lambda = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{\frac{\delta}{2\pi}}{\sqrt{1+\left(\frac{\delta}{2\pi}\right)^2}}}$$

$$\lambda = \frac{0{,}223}{\sqrt{1+0{,}05}} = \boxed{0{,}22 = \lambda}$$

$$T = 3{,}1 \cdot 200\,\mu s = \boxed{620\,\mu s = T}$$

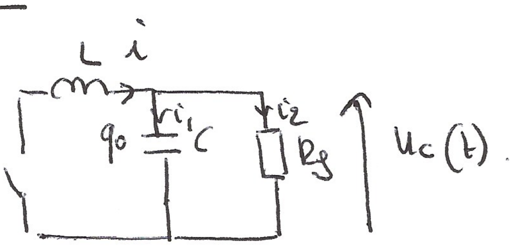
or $T = \frac{2\pi}{\omega_0\sqrt{1-\lambda^2}} \Rightarrow$
$$\left\{\begin{array}{l} \omega_0 = \frac{2\pi}{T\sqrt{1-\lambda^2}} \\ \frac{R_1+R_2}{R_2}\cdot\frac{1}{LC} = \frac{4\pi^2}{T^2(1-\lambda^2)} \end{array}\right.$$

$$\Rightarrow C = \frac{\frac{R_1+R_2}{R_2}\cdot\frac{1}{L}}{\frac{4\pi^2}{T^2(1-\lambda^2)}} = \boxed{\frac{R_1+R_2}{R_2}\cdot\frac{1}{L}\cdot\frac{T^2(1-\lambda^2)}{4\pi^2}}$$

AN $C = \frac{5200}{5000}\cdot\frac{1}{10^{-1}}\cdot\frac{(620)^2\cdot 10^{-12}}{4\pi^2}(1-0{,}22^2)$. $\Rightarrow$ $\boxed{C \simeq 105\,\mu F.}$

(2)

Ex3



$$\begin{cases} L\dfrac{di}{dt} + u_c = 0 & (1) \\ i_1 = C\dfrac{du_c}{dt} \; (2) \quad \text{et} \quad i = i_1 + i_2 \; (4) \\ i_2 = \dfrac{u_c}{R} & (3) \end{cases}$$

(4) ds (1) $\Rightarrow$ $L\dfrac{di_1}{dt} + L\dfrac{di_2}{dt} + u_c = 0$ A l'aide de 2 et 3

$$LC\frac{d^2u_c}{dt^2} + \frac{L}{R}\frac{du_c}{dt} + u_c = 0 \Rightarrow \boxed{\frac{d^2u_c}{dt^2} + \frac{1}{RC}\frac{du_c}{dt} + \frac{u_c}{LC} = 0.}$$

A comparer à la forme canonique $\dfrac{d^2u_c}{dt^2} + 2\lambda\omega_0\dfrac{du_c}{dt} + \omega_0^2 u_c = 0$

ou $\dfrac{d^2u_c}{dt^2} + \dfrac{\omega_0}{Q}\dfrac{du_c}{dt} + \omega_0^2 u_c = 0.$

donc $\boxed{\omega_0^2 = \dfrac{1}{LC}}$ et $\dfrac{\omega_0}{Q} = \dfrac{1}{RC} \rightarrow Q = \omega_0 \cdot RC = \dfrac{1}{\sqrt{LC}} \cdot RC$

$\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$ et $\boxed{Q = R\cdot\sqrt{\dfrac{C}{L}}}$

2°] à $t=0^-$ $q(0^-) = q_0 = q(0^+)$ et $u(0^+) = \dfrac{q(0^+)}{C} = \dfrac{q_0}{C}$. (Continuité de u donc de q aux bornes de C)

$i(0^-) = i(0^+)$ et $i(0^-) = 0$ (K ouvert)

continuité de i traversant la bobine. $\Rightarrow i(0^+) = 0$.

$$i_2(0^+) = \frac{u_c(0^+)}{R_g} = \frac{q_0}{R_g \cdot C}.$$

d'où $i_1(0^+) = i(0^+) - i_2(0^+) = -\dfrac{q_0}{R_g \cdot C} = C\dfrac{du_c}{dt}(0^+)$

$$\Rightarrow \frac{du_c(0^+)}{dt} = -\frac{q_0}{C}\cdot\frac{1}{R_g\cdot C} = \boxed{-\frac{q_0}{R_g C^2} = \frac{du_c}{dt}(0^+)}$$

3°] Régime pseudo périodique $u_c(t) = Ae^{-\frac{\omega_0}{2Q}t}\cos(\omega t + \varphi)$

avec $\omega = \omega_0\sqrt{1 - \dfrac{1}{4Q^2}}$

$$u_c(0^+) = \frac{q_0}{C} = A\cos\varphi.$$

$$\frac{du_c}{dt} = A\left[-\frac{\omega_0}{2Q} e^{-\frac{\omega_0}{2Q}t}\cos(\omega t+\varphi) - \omega e^{-\frac{\omega_0}{2Q}t}\sin(\omega t+\varphi)\right]$$

$$\frac{du_c}{dt}(0^+) = A\left[-\frac{\omega_0}{2Q}\cos\varphi - \omega\sin\varphi\right] = -\frac{q_0}{R_g\cdot C^2}$$

$$\cancel{\frac{q_0}{C}}\left[-\frac{\omega_0}{2Q} - \omega\,\mathrm{tg}\,\varphi\right] = -\frac{\cancel{q_0}}{R_g\cdot C^2}$$

$$\frac{\omega_0}{2Q} + \omega\,\mathrm{tg}\,\varphi = \frac{1}{R_g\cdot C} \rightarrow \boxed{\mathrm{tg}\,\varphi = \frac{\frac{\omega_0}{2Q} - \frac{1}{R_g\cdot C}}{\omega}}$$

et $\frac{1}{\cos^2\varphi} = 1+\mathrm{tg}^2\varphi \Rightarrow \frac{1}{\cos\varphi} = \sqrt{1+\left(\frac{\omega_0}{2Q\omega} - \frac{1}{R_g\cdot C\cdot\omega}\right)^2}$. $\left(\begin{array}{c}\varphi = \mathrm{Arctg}(\quad)\\ \cos\varphi > 0\end{array}\right)$

d'où $A = \frac{q_0}{C}\cdot\frac{1}{\cos\varphi}$.

$$u_c(t) = \frac{q_0}{C}\cdot\sqrt{1+\left(\frac{\omega_0}{2Q\omega} - \frac{1}{R_g\cdot C\omega}\right)^2}\cos(\omega t+\varphi).$$

4°] $\delta = \frac{1}{n}\ln\frac{u_c(t)}{u_c(t+nT)}$ $\qquad u_c(t) = 0{,}26\,V$ (2e sommet)

$u_c(t+3T) = 0{,}06\,V$

$$\Rightarrow \delta = \frac{1}{3}\ln\frac{u_c(t)}{u_c(t+3T)} = \boxed{0{,}49. = \delta}$$

or $$\delta = \frac{1}{n}\ln\left(\frac{A e^{-\frac{\omega_0}{2Q}t}\cos(\omega t+\varphi)}{A e^{-\frac{\omega_0}{2Q}(t+nT)}\underbrace{\cos(\omega(t+nT)+\varphi)}_{\cos \omega t+\varphi}}\right) = \frac{1}{n}\ln \exp\left(\frac{\omega_0}{2Q}\cdot nT\right)$$

$$= \frac{\cancel{\omega_0}}{2Q}\cdot\frac{2\pi}{\cancel{\omega_0}\sqrt{1-\frac{1}{4Q^2}}} = \boxed{\frac{2\pi}{\sqrt{4Q^2-1}} = \delta} \Rightarrow \frac{4\pi^2}{4Q^2-1} = \delta^2$$

$$4Q^2 = \frac{4\pi^2}{\delta^2}+1$$

$$\boxed{\varphi^2 = \frac{\pi^2}{\delta^2} + \frac{1}{4}} \Rightarrow \boxed{\varphi = 6{,}4}$$

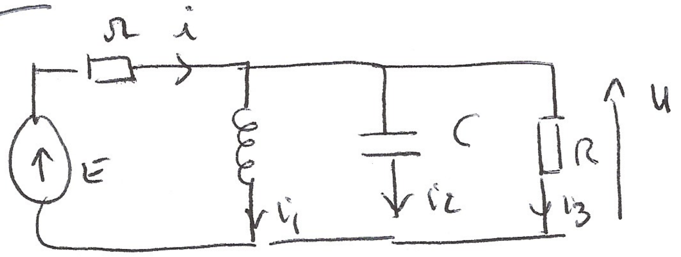
$$T = \frac{2\pi}{\omega_0\sqrt{1 - \frac{1}{4\varphi^2}}} = 4 \times 0{,}2.10^{-5}\,s = 0{,}8.10^{-5}\,s \Rightarrow \omega_0 = 7{,}87.10^{5}\,rad\,s^{-1}$$

or $\omega_0 = \dfrac{1}{\sqrt{LC}} \Rightarrow L = \dfrac{1}{\omega_0^2 . C} = \dfrac{1}{(7{,}87)^2 . 10^{10} . 5 . 10^{-9}} = 3{,}2.10^{-4}\,H.$

$$\boxed{L = 32\,mH.}$$

Enfin $\varphi = \dfrac{R}{\delta}\sqrt{\dfrac{L}{C}} \Rightarrow \boxed{R_g = 1{,}6.10^{3}\,\Omega}$

Ex4



$$\begin{cases} E = ri + u. & (1) \\ u = L\dfrac{di_1}{dt} = Ri_3 & (2) \\ i_2 = C\dfrac{du}{dt}. & (3) \\ i = i_1 + i_2 + i_3 & (4) \end{cases}$$

1°) à $t = 0^-$ $\quad i_1(0^-) = 0 \rightarrow i_1(0^+) = 0$

$u(0^-) = 0 \rightarrow u(0^+) = 0.$

(1) $\quad i(0^+) = \dfrac{E}{r}.$

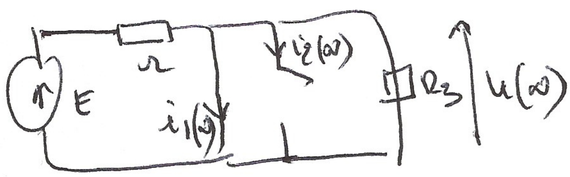
(2) $\quad i_3(0^+) = 0$

(4) $\Rightarrow i_2(0^+) = i(0^+) = \dfrac{E}{r}$

Rq $\quad \dfrac{di_3}{dt}(0^+) = \dfrac{1}{R}\dfrac{du}{dt}(0^+) = \dfrac{1}{RC}\, i_2(0^+) = \dfrac{E}{r.RC}$

2°) Régime permanent continu

Circuit équivalent.



$i_2(\infty) = 0$ (C = interrupteur ouvert)

$u(\infty) = 0$ (Tension aux bornes d'un fil = L)

$i_3(\infty) = 0$

$i_1(\infty) = i(\infty) = \dfrac{E}{r}$

3°) $u = Ri_3.$

or $E = r(i_1 + i_2 + i_3) + u.$

$E = r\left(i_1 + C\dfrac{du}{dt} + i_3\right) + u. \rightarrow$ On dérive et remplace u par Ri_3

$$0 = r\left[\frac{Ri_3}{L} + RC\frac{d^2i_3}{dt^2} + \frac{di_3}{dt}\right] + R\frac{di_3}{dt}$$

$$RC\frac{d^2i_3}{dt^2} + \left(1 + \frac{R}{r}\right)\frac{di_3}{dt} + \frac{R}{L}i_3 = 0 \Rightarrow \frac{d^2i_3}{dt^2} + \frac{\left(1+\frac{R}{r}\right)}{RC}\frac{di_3}{dt} + \frac{i_3}{LC} = 0$$

$$\boxed{\omega_0^2 = \frac{1}{LC}} \qquad 2\lambda\omega_0 = \frac{\left(1+\frac{R}{r}\right)}{R\cdot C} \implies \lambda = \frac{1+\frac{R}{r}}{RC}\cdot\frac{\sqrt{LC}}{2}.$$

$$\boxed{\lambda = \frac{1+\frac{R}{r}}{2R}\cdot\sqrt{\frac{L}{C}}.}$$

$$\frac{d^2 i_3}{dt^2} + 2\lambda\omega_0 \frac{di_3}{dt} + \omega_0^2 i_3 = 0$$

e^{rt} solution $\rightarrow$ $r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$\Delta = (4\lambda^2 - 4)\,\omega_0^2 < 0$ régime pseudo-périodique

$\implies \lambda < 1.$

$$\boxed{\frac{1+\frac{R}{r}}{2R}\sqrt{\frac{L}{C}} < 1}$$

5] $T = \dfrac{2\pi}{\omega_0\sqrt{1-\lambda^2}} = \boxed{\dfrac{2\pi\cdot\sqrt{LC}}{\sqrt{1-\dfrac{(1+R/r)^2}{4R^2}\cdot\dfrac{L}{C}}} = T.}$

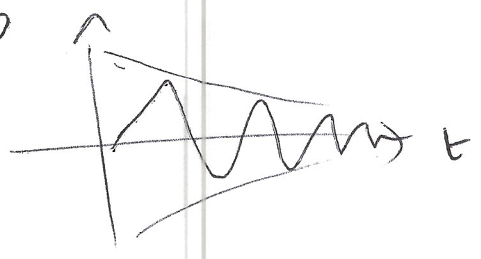
$$i_3(t) = A e^{-\lambda\omega_0 t}\left(\cos(\omega t + \varphi)\right).$$

$i_3(0^+) = 0 \implies 0 = A\cos\varphi \implies \varphi = \frac{\pi}{2}.$

$$\frac{di_3}{dt} = A\left[\cancel{\omega}\,\lambda\omega_0 e^{-\lambda\omega_0 t}\cos(\quad) - \omega e^{-\lambda\omega_0 t}\sin(\quad)\right].$$

$$\frac{di_3}{dt}(0^+) = A\left[-\lambda\omega_0\underbrace{\cos\varphi}_{0} - \omega\underbrace{\sin\varphi}_{1}\right] = \frac{E}{R\cdot rC}.$$

$$\implies A = -\frac{E}{R\cdot r\cdot C\omega}$$



$$\implies \boxed{i_3(t) = \frac{E}{R\cdot rC\omega}\, e^{-\lambda\omega_0 t}\,\cancel{\cos}\sin\omega t.}$$