Correction DS7 30/04/25

ex1

1°] $PV = nRT$ $n = \frac{N}{\mathcal{N}_a}$ ← nombre de molecules

nb de moles

$n^* = \frac{N}{V}$

$P = n^* \frac{R}{\mathcal{N}_a} \cdot T = n^* k_B \cdot T = \frac{1}{3} n^* \cdot m u^2 \Rightarrow \boxed{u = \sqrt{\frac{3 k_B \cdot T}{m}}}$

2°] $M = m \cdot \mathcal{N}_a \Rightarrow \boxed{m = \frac{M}{\mathcal{N}_a}}$ AN $m = \frac{40 \cdot 10^{-3}}{6{,}02 \cdot 10^{23}} =$

d'où $\boxed{u = 427 \, m s^{-1}}$

3°] a) $U_{mol} = (E_c)_{molaire} = \sum \frac{1}{2} m v_i^2 = \frac{1}{2} m \sum v_i^2 = \frac{1}{2} m \cdot \mathcal{N}_a u^2$

$= \frac{1}{2} \cancel{m} \cdot \mathcal{N}_a \cdot \frac{3 k_B \cdot T}{\cancel{m}} = \boxed{\frac{3}{2} RT = U_{mol}}$

b] $\frac{dU_{mol}}{dT} = \frac{3}{2} R = C_v$: capacité thermique à volume cst.

Variation d'energie interne quand T varie de 1 K.

ex2

I.A Atmosphère isotherme.

1) $PV = nRT_0 = \frac{m}{M_e} RT_0 \rightarrow \boxed{\mu = \frac{P M_e}{R T_0}}$

2] dz dz à eq $- \vec{grad} P \, d\tau + \mu \vec{g} \, d\tau = 0 \Rightarrow$

$\boxed{\vec{grad} P = \mu \vec{g}}$ projection sur oz ascendant $\Rightarrow \frac{dP}{dz} = -\mu g = -\frac{P M_e g}{R T_0}$

$\boxed{\frac{dP}{dz} + \frac{M g}{R T_0} \cdot P = 0}$ On pose $H = \frac{R T_0}{M_e g} \Rightarrow \boxed{P = P_0 e^{-\frac{z}{H}}}$

3] $M_{O_2} = 32 g \, mol^{-1}$ $M_e = 20\% \cdot M_{O_2} + 80\% \, M_{N_2} = 28{,}8 \cdot 10^{-3} \, kg \, mol^{-1}$.

$M_{N_2} = 28 g \, mol^{-1}$ $H = 8{,}5$ km.

$P = P_0/2 \Rightarrow -\frac{z_{50\%}}{H} = \frac{1}{2} \Rightarrow \boxed{z_{50\%} = H \cdot \ln 2}$ AN $\boxed{z_{50\%} = 5{,}9 \text{ km}.}$

B].

$$T = T_0(1-\alpha z)$$

4°)

$$\frac{dP}{dz} + \frac{Mg}{RT_0(1-\alpha z)} P = 0 \Rightarrow \frac{dP}{P} = -\frac{dz}{H(1-\alpha z)}$$

d'où $\ln P = \frac{1}{\alpha H}\ln(1-\alpha z) + cste \Rightarrow \ln\frac{P}{P_0} = \ln(1-\alpha z)^{\frac{1}{\alpha H}}$

$\ln P_0 = 0 + cste$

$$\Rightarrow \boxed{P = P_0(1-\alpha z)^{\beta}} \text{ avec } \boxed{\beta = \frac{1}{\alpha H} = \frac{z_0}{H}}$$

La loi des G.P donne :

$$\mu = \frac{P.M}{R.T} = \frac{P_0(1-\alpha z)^{\beta}.M}{RT_0(1-\alpha z)} \Rightarrow \boxed{\mu = \mu_0(1-\alpha z)^{\beta-1}}$$

5°) $P(z) = \frac{P_0}{2} \Rightarrow \frac{1}{2} = (1-\alpha z)^{\beta} = 2^{-\frac{1}{\beta}} - 1 = -\alpha z_{50\%} \Rightarrow \boxed{z_{50\%} = z_0\left(1-2^{-\frac{1}{\beta}}\right)}$

$\Rightarrow \boxed{z_{50\%} = 5{,}4 \text{ km}}$ Valeur proche de celle du modèle isotherme mais inférieure car T ↘ lorsque z ↑.

6°]. fig 2 → T fonction linéaire et décroissante de z.

$T(z=0) \simeq 288K$.

$$T = T_0 - T_0\alpha.z \Rightarrow T_0.\alpha = 6{,}94 \Rightarrow$$
$$= 288{,}14 - 6{,}94.z$$

$$\alpha = \frac{1}{z_0} = \frac{6{,}94}{288} \Rightarrow$$

$$\boxed{z_0 = \frac{288}{6{,}94} = 41{,}5 \text{ km}}$$

$$\left.\begin{array}{l} P = P_0(1-\alpha z)^{\beta} \\ T = T_0(1-\alpha z)\end{array}\right\} \Rightarrow P = P_0\left(\frac{T}{T_0}\right)^{\beta}$$

$$= 1{,}01\left(\frac{T}{288{,}08}\right)^{5{,}25} \Rightarrow P_0 = 1{,}01 \text{ bar}.$$

$$\beta = 5{,}25 = \frac{z_0}{H} \Rightarrow \boxed{H = 7{,}9 \text{ km}}$$

Valeurs proches.

7°) Atmosphère : $(P_e ; \mu_e ; T_e)$.

Volume du ballon $V_0 = 2000\ m^3$

m = masse nacelle + passagers : $= 500\ kg$.

m_i = masse d'air chaud.

Intérieur du ballon $(P_i ; T_i)$. $P_i = P_e$.

$P_i V_0 = \frac{m_i}{M_e} R T_i$ et $P_i = P_e$ $\quad m_i = \frac{P_e V_0}{R T_i} \cdot M_e$ et $\mu_e = \frac{P_e \cdot M_e}{R T_e}$.

$$\boxed{m_i = \frac{T_e}{T_i} V_0 \cdot \mu_e}$$

8°) A l'éq $\vec{F}_{arch} + \vec{P}_{oids} = 0$; projection sur z ascendant.

$\mu_e \cdot \cancel{g} \cdot V_0 - (m + m_i)\cancel{g} = 0$. et $m = \mu_e V_0 - m_i$

et $\mu_e \cdot V_0 = m_i \cdot \frac{T_i}{T_e}$ $\Rightarrow$ $\boxed{m = m_i \left(\frac{T_i}{T_e} - 1\right)}$

9°) en $z = z_m$ $\quad \vec{F}_{arch} - m\vec{g} = 0$

$\mu_e V_0 g = mg$ $\Rightarrow$ $m = V_0 \mu_0 (1 - \alpha z_m)^{\beta - 1}$

$\Rightarrow$ $\boxed{z_m = \frac{1}{\alpha}\left(1 - \left(\frac{m}{\mu_0 V_0}\right)^{\frac{1}{\beta - 1}}\right)}$ $\quad$ AN $\boxed{z_m = 13\ km}$

10°) Calcul de T_i min par l'éq. mécanique au sol : $\quad T_e = T_0$

$m + m_i < \mu_e V_0$ $\Rightarrow$ $m + \frac{T_e}{T_i} V_0 \mu_e < \mu_e V_0$. avec $\mu_e = \mu_0$.

$m < \mu_e V_0 \left(1 - \frac{T_0}{T_i}\right)$ $\Rightarrow$ $1 - \frac{T_0}{T_i} > \frac{m}{\mu_0 V_0}$ $\Rightarrow$ $\frac{T_0}{T_i} < 1 - \frac{m}{\mu_0 V_0}$.

$T_d > \frac{T_0}{1 - \frac{m}{V_0 \mu_0}}$ $\Rightarrow$ $\boxed{T_d = \frac{T_0}{1 - \frac{m}{V_0 \mu_0}}}$ $\quad$ AN $T_d \geq 362\ K$.

Eq en $z \neq 0$ $\quad m + \frac{T_e}{T_i} v_0 \mu_e = \mu_e v_0$.

Eq en $z=0$ $\quad m + \frac{T_0}{T_d} v_0 \mu_0 = \mu_0 v_0$.

d'où en égalant les 2 valeurs de m :

$$\mu_e\left(1-\frac{T_e}{T_i}\right) = \mu_0\left(1-\frac{T_0}{T_d}\right)$$

et $\mu_e = \mu_0(1-\alpha z)^{\beta-1}$

$P_e = P_0(1-\alpha z)^{\beta}$

$T_e = T_0(1-\alpha z)$.

$$\mu_0 \frac{(1-\alpha z)^\beta}{(1-\alpha z)}\left(1-\frac{T_e}{T_i}\right) = \mu_0\left(1-\frac{T_0}{T_d}\right).$$

$$\cancel{\mu_0}\frac{P_e}{P_0(1-\alpha z)}\left(1-\frac{T_e}{T_i}\right) = \cancel{\mu_0}\left(1-\frac{T_0}{T_d}\right)$$

$$\frac{P_e \cdot T_0}{P_0 \; T_e}\left(1-\frac{T_e}{T_i}\right) = 1-\frac{T_0}{T_d} \Rightarrow \boxed{P_e\left(\frac{1}{T_e}-\frac{1}{T_i}\right) = P_0\left(\frac{1}{T_0}-\frac{1}{T_d}\right)}$$

ex 3.

ex 3

36) $\left.\begin{array}{l} C_p - C_v = nR \\ \gamma = \frac{C_p}{C_v}\end{array}\right\} \to \begin{array}{l} C_v = \frac{nR}{\gamma-1} \\ C_p = \frac{nR\gamma}{\gamma-1}\end{array}$ or $P_0 V_A = nRT_0 \Rightarrow \boxed{C_v = \frac{P_0 V_A}{T_0(\gamma-1)}}$ $\boxed{C_p = \frac{P_0 V_A \cdot \gamma}{T_0(\gamma-1)}}$

37) de $0 \to 1$ la cale reste en contact avec A donc le volume ne varie pas. $\boxed{\text{La transformation est isochore}}$.

38) Le piston est à l'équilibre ds l'état 1 : $\vec{P}_{piston} + \vec{F}_{gaz/piston} + \vec{F}_{atm/piston} = \vec{0}$

$\Rightarrow (mg - P_1 S + P_0 S)\vec{e_z} = \vec{0} \Rightarrow \boxed{P_1 = P_0 + \frac{mg}{S}}$.

$\begin{array}{l} P_0 V_A = nRT_0 \\ P_1 V_A = nRT_1 \end{array} \Rightarrow \frac{P_1}{P_0} = \frac{T_1}{T_0} \to T_1 = T_0 \cdot \frac{P_1}{P_0} \Rightarrow \boxed{T_1 = T_0 \frac{\left(P_0 + \frac{mg}{S}\right)}{P_0}}$

AN $T_1 = \cancel{330}$ K

330 K.

39) $\Delta U_{0\to1} = Q_{0\to1} + \underbrace{W_{0\to1}}_{=0}$ car la transformation est isochore.

$$Q_{0\to1} = n C_v (T_1 - T_0) = \frac{nR}{\gamma-1}(T_1 - T_0) = \boxed{\frac{p_0 V_A}{\gamma-1}\left(\frac{T_1}{T_0} - 1\right) = Q_{0\to1}}$$

AN $\boxed{Q_{0\to1} = 8{,}25\ J}$.

40) de 1 à 2, $\boxed{p_{gaz} = p_1 = p_0 + \frac{mg}{S} = p_2}$ (On chauffe très doucement donc les variables d'état restent définies.) $\Rightarrow$ $\boxed{\text{Transformation isobare}}$

41)
$$p_1 V_A = nRT_1$$
$$p_1 V_B = nRT_2$$
$$\Rightarrow \frac{T_2}{T_1} = \frac{V_B}{V_A} \Rightarrow \boxed{T_2 = T_1 \cdot \frac{V_B}{V_A}}$$

AN $\boxed{T_2 = 1000\ K}$.

42) $$\Delta H_{1\to2} = Q_{1\to2} = n C_p (T_2 - T_1) = \frac{nR\gamma}{\gamma-1}\Delta T = \boxed{\frac{p_0 V_A \cdot \gamma}{T_0(\gamma-1)}(T_2 - T_1)}$$

AN $\boxed{Q_{1\to2} = 260\ J}$.

43) 2→3 La cale reste en B La transformation est isochore quasistatique

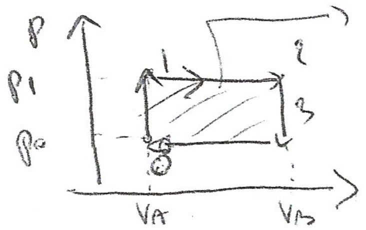
et $p_3 = p_0$ et $p_3 V_B = p_0 V_B = nRT_3$. $\Rightarrow \frac{T_3}{T_2} = \frac{p_0}{p_2} \Rightarrow$
$p_2 V_B = nRT_2$

$$\boxed{T_3 = T_2 \cdot \frac{p_0}{p_1}}$$

3→0 Transformation isobare à $p_3 = p_0$.

44) $$W_{cycle} = W_{0\to1} + W_{1\to2} + W_{2\to3} + W_{3\to0}.$$
$$= 0 + -p_1(V_B - V_A) + 0 - p_0(V_A - V_B) = V_B(p_0 - p_1) + V_A(p_1 - p_0)$$
$$= (V_B - V_A)[p_0 - p_1] = \boxed{-\frac{mg}{S}(V_B - V_A) =}$$

AN $\boxed{W = -6{,}7\ J}$ Rq $W_{cycle} < 0 \Rightarrow$ cycle moteur

45+46) 

Aire $= |W_{cycle}|$

cycle décrit ds le sens horaire donc $\boxed{W < 0}$ et

Aire du cycle $= (p_1 - p_0)\cdot(V_B - V_A)$.

$$\eta = \frac{-W}{Q_{0\to1}}$$

I] Préliminaires.

1) $H_s = U_s + P.V$

2) $\Delta H_s = \Delta U_s + \Delta(PV) = \Delta U_s + P_{ext}\Delta V$ puisque $P_s = P_{ext}$.

or $\Delta H_s = W_s + Q_{ext}$ et $W_s = -P_{ext}\Delta V$

d'où $\boxed{\Delta H_s = Q_{ext}}$.

3) $\boxed{dH_s = mc\,dT = \delta Q_{ext}}$.

II Le récipient est thermiquement isolé.

à $t=0$ $T = T_{ext}$.

1) $\boxed{\Delta H = Q_{ext} = mc(T_1 - T_{ext}) > 0.}$

2) or $\cancel{\Delta H}$ $Q_{ext} = P_{th}.\Delta t$.

d'où $\boxed{\Delta t = \dfrac{mc(T_1 - T_{ext})}{P_{th}}}$

AN $m = 100$ kg.
$c = 4{,}18.10^3\,J\,kg^{-1}\,K^{-1}$
$T_{ext} = 290\,K$
$T_1 = 350\,K$
$P_{th} = 1{,}5.10^3\,W$

$Q_{ext} = 100.4{,}18.10^3(350-290) = 4{,}18.60.10^5 = 2{,}5.10^7\,J$.

$\Delta t = \dfrac{2{,}5.10^7}{1{,}5.10^3} = 16666\,s = 4{,}6\,h$

III Pertes thermiques.

$\delta Q_p = -k(T - T_{ext})\,dt$

eau $T \Rightarrow T_{ext}$.
$\delta Q_p < 0$ si $T > T_{ext}$

or $\delta Q_p = -k\underbrace{(T - T_{ext})}_{>0}\underbrace{dt}_{>0} \Rightarrow \boxed{k > 0}$

) En régime forcé $dH_{eau} = 0$ puisque $T = T_{lim} = cte$.

d'où $dH_{eau} = P_{th}\,dt - k(T_{lim} - T_{ext})\,dt = 0$.

$\Rightarrow \boxed{k = \dfrac{P_{th}}{T_{lim} - T_{ext}}}$

AN $T_{ext} = 290\,K$ $P_{th} = 1{,}5.10^3\,W$.
$T_{lim} = 370\,K$

$\Rightarrow k = \dfrac{1{,}5.10^3}{80} = \boxed{18{,}8\,W\,K^{-1} = k.}$

(3)

2) Le chauffage est arrêté.

à $t=0$, $T = T_{lim}$.

2) $dH_{eau} = mc\,dT = -k(T - T_{ext})\,dt$.

$\Rightarrow mc\dfrac{dT}{dt} + kT = kT_{ext}$. $\Rightarrow \dfrac{dT}{dt} + \dfrac{k}{mc}T = \dfrac{k}{mc}T_{ext}$.

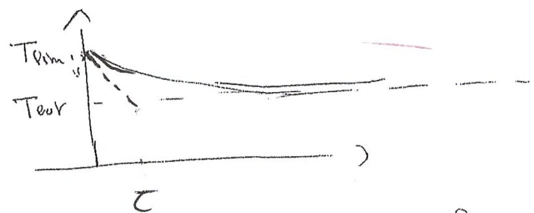
$\tau = \dfrac{mc}{k}$.

$T(t) = Ae^{-\left(\frac{k}{mc}\right).t} + T_{ext}$

à $t=0$ $T_{lim} = A + T_{ext} \Rightarrow A = T_{lim} - T_{ext}$. $\Rightarrow$.

$\boxed{T(t) = (T_{lim} - T_{ext})\,e^{-\frac{k}{mc}t} + T_{ext}.}$

$t \to \infty$ $T \to T_{ext}$.



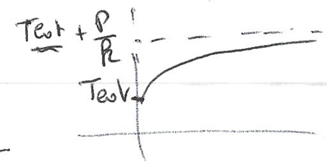
3°] Montée en température de la masse d'eau.

$t=0$ $T = T_{ext}$.

$mc\,dT = P\,dt - k(T - T_{ext})\,dt$.

$\Rightarrow mc\dfrac{dT}{dt} + kT = P + kT_{ext}$. $\Rightarrow \boxed{\dfrac{dT}{dt} + \dfrac{k}{mc}T = \dfrac{P}{mc} + \dfrac{k}{mc}T_{ext}}$

$T(t) = Ae^{-\frac{k}{mc}t} + \dfrac{P}{k} + T_{ext}$.



à $t=0$ $T(t) = T_{ext} = A + \dfrac{P}{k} + T_{ext} \Rightarrow$ $A = -\dfrac{P}{k}$.

$\boxed{T(t) = -\dfrac{P}{k}e^{-\frac{k}{mc}t} + \dfrac{P}{k} + T_{ext}.}$

pour $\Delta t' = t_1$, $T = T_1$.

$T_1 = -\dfrac{P}{k}e^{-\frac{k}{mc}t_1} + \dfrac{P}{k} + T_{ext}$.

$\Rightarrow \left(T_1 - T_{ext} - \dfrac{P}{k}\right)\times -\dfrac{k}{P} = e^{-\frac{k}{mc}t_1}$

$1 - \dfrac{k}{P}(T_1 - T_{ext}) = e^{-\frac{k}{mc}t_1}$ $\Rightarrow$ (4)

$\boxed{t_1 = -\dfrac{mc}{k}\ln\left(1 - \dfrac{k}{P}(T_1 - T_{ext})\right).}$

AN $t_1 = 8{,}6\,h$.