# Développements limités usuels

Les formules ci-dessous sont à connaître ou à savoir retrouver rapidement :

$$
\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \underset{x\to 0}{o}(x^n)\\
\mathrm{sh}(x) &= x + \frac{x^3}{6} + \frac{x^5}{120} + \cdots + \frac{x^{2k+1}}{(2k+1)!} + \underset{x\to 0}{o}(x^{2k+2})\\
\mathrm{ch}(x) &= 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2k}}{(2k)!} + \underset{x\to 0}{o}(x^{2k+1})
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{1+x} &= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \underset{x\to 0}{o}(x^n)\\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \underset{x\to 0}{o}(x^n) \quad (x > -1)
\end{aligned}
$$

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\begin{aligned}
\sin(x) &= x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots + (-1)^k\frac{x^{2k+1}}{(2k+1)!} + \underset{x\to 0}{o}(x^{2k+2})\\
\cos(x) &= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots + (-1)^k\frac{x^{2k}}{(2k)!} + \underset{x\to 0}{o}(x^{2k+1})\\
\tan(x) &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \underset{x\to 0}{o}(x^6)
\end{aligned}
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\begin{aligned}
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3 + \cdots\\
&\quad + \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}x^n + \underset{x\to 0}{o}(x^n) \quad (\alpha \in \mathbb{R})\\
\sqrt{1+x} &= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \underset{x\to 0}{o}(x^3)\\
\sqrt[3]{1+x} &= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 + \underset{x\to 0}{o}(x^3)
\end{aligned}
$$

$$
\begin{aligned}
\arctan(x) &= x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^k\frac{x^{2k+1}}{2k+1} + \underset{x\to 0}{o}(x^{2k+2})\\
\arcsin(x) &= x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots + \frac{(2k)!}{2^{2k}(k!)^2(2k+1)}x^{2k+1} + \underset{x\to 0}{o}(x^{2k+2})
\end{aligned}
$$