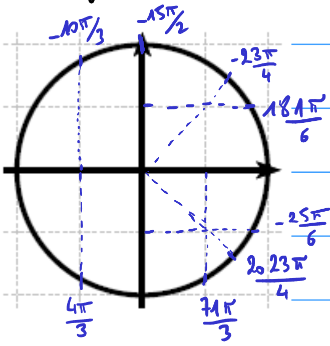
Correction IE trigo générale.

Exercice 1



$-\frac{23\pi}{4} = -\frac{24\pi}{4} + \frac{\pi}{4} = -6\pi + \frac{\pi}{4} \equiv \frac{\pi}{4}\ [2\pi]$

$-\frac{10\pi}{3} = -\frac{6\pi}{3} - \frac{4\pi}{3} = -2\pi - \frac{4\pi}{3} \equiv \frac{2\pi}{3}\ [2\pi]$

$\frac{2023\pi}{4} = \frac{2024\pi}{4} - \frac{\pi}{4} \equiv 506\pi - \frac{\pi}{4} \equiv -\frac{\pi}{4}\ [2\pi]$

$\frac{71\pi}{3} = 12 \times \frac{6\pi}{3} - \frac{\pi}{3} \equiv -\frac{\pi}{3}\ [2\pi]$

$-\frac{15\pi}{2} = -\frac{16\pi}{2} + \frac{\pi}{2} = -8\pi + \frac{\pi}{2} \equiv \frac{\pi}{2}\ [2\pi]$

$-\frac{25\pi}{6} = -2 \times \frac{12\pi}{6} - \frac{\pi}{6} \equiv -\frac{\pi}{6}\ [2\pi]$

$\frac{181\pi}{6} = \frac{180\pi}{6} + \frac{\pi}{6} = 30\pi + \frac{\pi}{6} \equiv \frac{\pi}{6}\ [2\pi]$

Ex 2 :

$A = \sin x - \cos x + 2\sin x = 3\sin x - \cos x$

$B = \sin\left(\frac{\pi}{8}\right) - \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} - \sin\frac{7\pi}{8} = \sin\frac{\pi}{8} - \sin\frac{3\pi}{8} + \sin\left(\pi - \frac{3\pi}{8}\right) - \sin\left(\pi - \frac{\pi}{8}\right)$

$= \cancel{\sin\frac{\pi}{8}} - \cancel{\sin\frac{3\pi}{8}} + \sin\frac{3\pi}{8} - \sin\frac{\pi}{8} = 0$

Ex 3 :

1) $(\vec{AE}, \vec{AC}) \equiv \frac{\pi}{4} + \frac{\pi}{3} \equiv \frac{7\pi}{12}\ [2\pi]$ MP : $\frac{7\pi}{12}$

2) $(\vec{AD}; \vec{EA}) \equiv (\vec{AD}; \vec{AE}) + \pi \equiv -\frac{\pi}{4} - \frac{\pi}{3} - \frac{\pi}{4} + \pi \equiv \frac{\pi}{6}\ [2\pi]$. MP : $\frac{\pi}{6}$

3) $(\vec{AE}, \vec{BC}) \equiv (\vec{AE}, \vec{EB}) + (\vec{EB}, \vec{BC}) \equiv (\vec{EA}, \vec{EB}) + \cancel{\pi} + (\vec{BE}, \vec{BC}) + \cancel{\pi}\ [2\pi]$

$\equiv -\frac{\pi}{2} - \frac{\pi}{4} - \frac{\pi}{3}\ [2\pi]$

$\equiv -\frac{3\pi}{4} - \frac{\pi}{3} \equiv -\frac{13\pi}{12}\ [2\pi]$ MP : $\frac{11\pi}{12}$

4°) $(\vec{BE}, \vec{CD}) \equiv (\vec{BE}, \vec{BC}) + (\vec{BC}, \vec{CD}) \equiv -\frac{\pi}{4} - \frac{\pi}{3} + (\vec{CB}, \vec{CD}) + \pi\ [2\pi]$

$\equiv -\frac{7\pi}{12} + \left(-\frac{\pi}{3} - \frac{\pi}{4}\right) + \pi\ [2\pi]$

$\equiv -\frac{14\pi}{12} + \pi \equiv -\frac{\pi}{6}\ [2\pi]$ MP : $-\frac{\pi}{6}$

Exercice 4

1°) $\sin x = -\frac{1}{2} \Leftrightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow x \equiv -\frac{\pi}{6}\ [2\pi]$ ou $x \equiv \frac{7\pi}{6}\ [2\pi]$

$S_{\mathbb{R}} = \left\{-\frac{\pi}{6} + k \times 2\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{7\pi}{6} + k \times 2\pi, k \in \mathbb{Z}\right\}$ - $S_{]-\pi,\pi]} = \left\{-\frac{5\pi}{6}; -\frac{\pi}{6}\right\}$

2°) $\cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{4} \Leftrightarrow x \equiv -\frac{\pi}{4}\ [2\pi]$ ou $x \equiv \frac{\pi}{4}\ [2\pi]$

$S_{\mathbb{R}} = \left\{\frac{\pi}{4} + k2\pi, k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}\right\}$ - $S_{]-\pi,\pi]} = \left\{-\frac{\pi}{4}; \frac{\pi}{4}\right\}$

3°) $\cos\left(3x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(3x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right) \Leftrightarrow 3x + \frac{\pi}{4} \equiv \frac{\pi}{6}\ [2\pi]$ ou $3x + \frac{\pi}{4} \equiv -\frac{\pi}{6}\ [2\pi]$

$\Leftrightarrow 3x \equiv -\frac{\pi}{12}\ [2\pi]$ ou $3x \equiv -\frac{5\pi}{12}\ [2\pi]$

$\Leftrightarrow x \equiv -\frac{\pi}{36}\left[\frac{2\pi}{3}\right]$ ou $x \equiv -\frac{5\pi}{36}\left[\frac{2\pi}{3}\right]$

$S_{\mathbb{R}} = \left\{-\frac{\pi}{36} + \frac{2k\pi}{3}, k \in \mathbb{Z}\right\} \cup \left\{-\frac{5\pi}{36} + \frac{k2\pi}{3}, k \in \mathbb{Z}\right\}$ - $S_{]-\pi,\pi]} = \left\{-\frac{25\pi}{36}, -\frac{\pi}{36}, \frac{23\pi}{36}, -\frac{29\pi}{36}, -\frac{5\pi}{36}; \frac{19\pi}{36}\right\}$

4°) $\cos 4x = \sin\left(2x + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2} - 2x - \frac{\pi}{4}\right) \iff 4x \equiv \frac{\pi}{4} - 2x\, [2\pi]$ ou $4x \equiv 2x - \frac{\pi}{4}\, [2\pi]$

$\iff x \equiv \frac{\pi}{24} \left[\frac{\pi}{3}\right]$ ou $x \equiv -\frac{\pi}{8}\, [\pi]$.

$S_{\mathbb{R}} = \left\{\frac{\pi}{24} + k\frac{\pi}{3},\ k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{8} + k\pi\right\}$. $S_{]-\pi,\pi]} = \left\{-\frac{23\pi}{24} ; -\frac{15\pi}{24} ; -\frac{7\pi}{24} ; \frac{\pi}{24} ; \frac{9\pi}{24} ; \frac{17\pi}{24} ; -\frac{\pi}{8} ; \frac{7\pi}{8}\right\}$

5°) $2\sin^2 x + \sin x - 1 = 0 \iff 2X^2 + X - 1 = 0 \iff (X+1)(2x-1) = 0$

$\iff \sin x = \frac{1}{2}$ ou $\sin x = -1$

$S_{\mathbb{R}} = \left\{\frac{\pi}{6} + k2\pi,\ k \in \mathbb{Z}\right\} \cup \left\{\frac{5\pi}{6} + k \times 2\pi,\ k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{2} + k \times 2\pi,\ k \in \mathbb{Z}\right\}$. $S_{]-\pi,\pi]} = \left\{\frac{5\pi}{6} ; \frac{\pi}{6} ; -\frac{\pi}{2}\right\}$

<u>Exercice 5</u> $(\overrightarrow{AE}, \overrightarrow{AB}) = \frac{2\pi}{3}\, [2\pi]$ $(\overrightarrow{BA}, \overrightarrow{AD}) = -\frac{\pi}{4}\, [2\pi]$ et $(\overrightarrow{AC}, \overrightarrow{AD}) \equiv \frac{5\pi}{12}\, [2\pi]$.

$(\overrightarrow{AE}, \overrightarrow{AC}) \equiv (\overrightarrow{AE}, \overrightarrow{AB}) + (\overrightarrow{AB}, \overrightarrow{AD}) + (\overrightarrow{AD}, \overrightarrow{AC})\, [2\pi]$

$\equiv \frac{2\pi}{3} + (\overrightarrow{BA}, \overrightarrow{AD}) + \pi - (\overrightarrow{AC}, \overrightarrow{AD})\, [2\pi]$

$\equiv \frac{2\pi}{3} - \frac{\pi}{4} + \pi - \frac{5\pi}{12}\, [2\pi]$

$\equiv \frac{8\pi}{12} - \frac{8\pi}{12} + \pi\, [2\pi] \equiv \pi\, [2\pi]$ donc E, A et C sont alignés (dans cet ordre)

<u>Exercice 6</u>

1°) Soit $x \in \mathbb{R}$. $\sin\left(x + \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3} - x\right) = \cancel{\sin\frac{\pi}{3}\cos x} + \cos\frac{\pi}{3}\sin x - \left(\cancel{\sin\frac{\pi}{3}\cos x} - \cos\frac{\pi}{3}\sin x\right)$

$= 2 \times \frac{1}{2}\sin x = \sin x$

2°) a) $\sin(a+b)\sin(a-b) = (\sin a \cos b + \sin b \cos a)(\sin a \cos b - \sin b \cos a)$

$= \sin^2 a \cos^2 b - \sin^2 b \cos^2 a = \sin^2 a (1 - \sin^2 b) - \sin^2 b (1 - \sin^2 a)$

$= \sin^2 a - \sin^2 b - \cancel{\sin^2 a \sin^2 b} + \cancel{\sin^2 b \sin^2 a}$

b) $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$ et $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

Donc $\sin\left(\frac{7\pi}{12}\right) \times \sin\left(\frac{\pi}{12}\right) = \sin^2\frac{\pi}{3} - \sin^2\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$