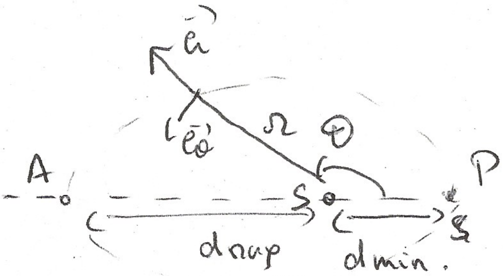
Corrigé TD forces centrales (2).

ex1.



$$r = \frac{p}{1+e\cos\theta}.$$

Données $T = 76$ ans.

$d_{min} = 0{,}59$.

$$\begin{cases} \dfrac{T^2}{a^3} = \dfrac{4\pi^2}{\mathcal{G} M_S} \Rightarrow a = \left(\dfrac{T^2 \cdot \mathcal{G} \cdot M_S}{4\pi^2}\right)^{1/3} \\ d_{min} = \dfrac{p}{1+e} \qquad d_{max} = 2a - d_{min} = \dfrac{p}{1-e}. \end{cases}$$

or $\dfrac{d_{max}}{d_{min}} = \dfrac{1+e}{1-e} \Rightarrow \boxed{e = \dfrac{d_{max} - d_{min}}{d_{max} + d_{min}}}$ $\boxed{p = (1+e)\, d_{min}.}$

<u>AN</u> $\mathcal{G} = 6{,}67 \cdot 10^{-11}\ N \cdot m^2 kg^{-2}$.

$$\begin{cases} d_{max} = 1{,}1 \cdot 10^{2}\ u.a = 5{,}3 \cdot 10^{12}\ m. \\ e = 0{,}97. \\ p = 1{,}1\ u.a. \end{cases}$$

ex2. Modèle de Bohr.

1) $p \circ \xleftarrow{\vec{F}} e$, r

$$\vec{F} = -\frac{e^2}{4\pi\varepsilon r^2}\vec{e_r} \rightarrow \delta W = \vec{F} \cdot d\vec{r} = -\frac{e^2}{4\pi\varepsilon r^2} dr = -dE_p.$$

$$\boxed{E_p(r) = -\frac{e^2}{4\pi\varepsilon r}.}$$

2) $m\vec{a} = \vec{F}$

3) $-m\dfrac{v^2}{r}\vec{e_r} = -\dfrac{e^2}{4\pi\varepsilon r^2}\vec{e_r} \rightarrow \boxed{v = \dfrac{e}{\sqrt{4\pi\varepsilon m \cdot r}}.}$

$$E_m = \frac{1}{2} m v^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \boxed{-\frac{e^2}{8\pi\varepsilon_0 r} = E_m.} \boxed{= + \frac{E_p}{2}.}$$

4°) $\vec{L_0}(M) = \vec{OM} \wedge m\vec{v} = m r v \vec{e_z} = m r . \frac{e}{\sqrt{4\pi\varepsilon_0 m r}} = e\sqrt{\frac{m . r}{4\pi\varepsilon_0}}$

$$\boxed{L_0 = e\sqrt{\frac{m . r}{4\pi\varepsilon_0}}.}$$

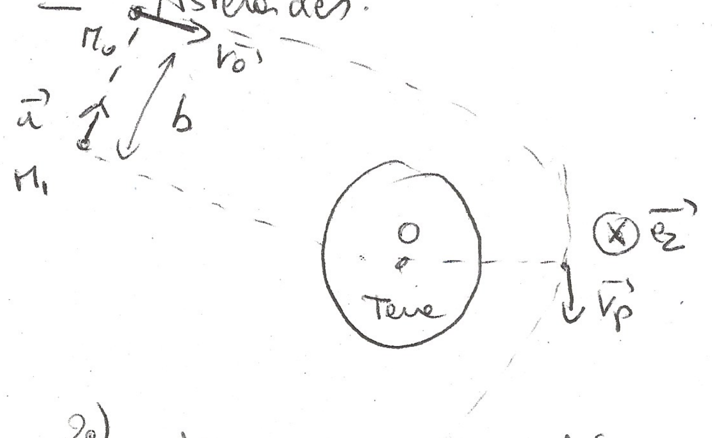
5°] Quantification $L_0 = e\sqrt{\frac{m r_n}{4\pi\varepsilon_0}} = n\hbar$

$$\boxed{r_n = \frac{4\pi\varepsilon_0 \hbar^2}{m . e^2} . n^2 .}$$

6°] $E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0} . \left(\frac{4\pi\varepsilon_0 . \hbar^2}{m e^2} . n^2\right)^{-1}.$

$$E_n = -\frac{e^2 . m e^2}{8\pi\varepsilon_0 . 4\pi\varepsilon_0 \hbar^2} . \frac{1}{n^2} = -\frac{E_0}{n^2}$$ avec $\boxed{E_0 = \frac{m e^4}{32\pi^2\varepsilon_0^2 \hbar^2} = 13{,}6 \text{ eV}}$

Ex 3 **Astéroïdes.**



$OM_0 \to \infty$

1°) $E_m = \frac{1}{2} m v_0^2 - \mathcal{G}\frac{m M_T}{OM_0} = \frac{1}{2} m v_0^2 > 0$

Trajectoire hyperbolique.

2°) $dE_c = -dE_p \Rightarrow d(E_c + E_p) = 0 \Rightarrow E_m = E_c + E_p = \text{cste}$.

d'où $\boxed{\frac{1}{2} m v_0^2 = \frac{1}{2} m v_p^2 - \mathcal{G}\frac{m M_T}{r_p}}$ (1)

3°) $\vec{L_O}(M) = \vec{OM} \wedge m\vec{v}$ TMC : $\frac{d\vec{L_O}}{dt} = \vec{0} \to \vec{L_O} = \vec{\text{cste}} \Rightarrow$

Rappel a) $\vec{L_O} = m r^2 \dot\theta \vec{e_z} = m C \vec{e_z}$

b) mvt plan.

c) loi des Aires.

$\vec{L_O}(P) = m r_p v_p \vec{e_z}$ car $\vec{v_p} \perp \vec{OP}$.

$\vec{L_O}(M_0) = \vec{OM_0} \wedge m\vec{v_0} = (\vec{OM_1} + b\vec{u}) \wedge m\vec{v_0}$ expression préférable

car $\|\vec{OM_1}\| \to \infty$ mais $\vec{OM_1} \wedge \vec{v_0} = \vec{0}$ car les 2 vecteurs sont colinéaires

alors que $\|\vec{OM_0}\| \to \infty$ et $\vec{OM_0} \wedge m\vec{v_0}$ impossible à évaluer.

d'où $\boxed{r_p v_p = b v_0}$ (2)

(1) et (2) : 2 équations à 2 inconnues r_p et v_p.

On élimine v_p. (2) $\Rightarrow v_p = \frac{b v_0}{r_p}$.

(1) devient : $\frac{1}{2} \cancel{m} v_0^2 = \frac{1}{2} \cancel{m} \frac{b^2 v_0^2}{r_p^2} - \mathcal{G}\frac{m M_T}{r_p}$.

$$v_0^2 r_p^2 + 2 G m_T \cdot r_p - b_0^2 v_0^2 = 0 .$$

$$\Delta = 4 G^2 m_T^2 + 4 b_0^2 v_0^4$$

$$r_p = \frac{- G m_T \pm \sqrt{G^2 m_T^2 + b_0^2 v_0^4}}{v_0^2}$$ Seule la racine positive

convient :

$$r_p = \frac{- G m_T + \sqrt{G^2 m_T^2 + b_0^2 v_0^4}}{v_0^2} = \boxed{- \frac{G m_T}{v_0^2} + \sqrt{b_0^2 + \left(\frac{G m_T}{v_0^2}\right)^2} = r_p}$$

AN
$$\begin{cases} G = 6{,}67 \cdot 10^{-11} \ N \cdot m^2 kg^{-2} \\ m_T = 6 \cdot 10^{24} \ kg . \\ v_0 = 2 \cdot 10^3 \ m \cdot s^{-1} \\ b_0 = 1{,}0 \cdot 10^8 \ m . \end{cases}$$