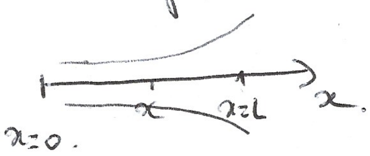
Correction DS5 PCSI2 08/03/24.

Ex1 Trompette.



1°] $P_i(x=0,t) = P_0\cos(\omega t+\varphi_i)$

2°] $P_i(x,t) = P(x=0, t-\tau)$ avec $\tau = \dfrac{x}{c}$.

$= P_0\cos(\omega(t-\tau)+\varphi_i) = P_0\cos\left(\omega t - \dfrac{2\pi \cdot x}{c\cdot T} + \varphi_i\right)$

$$\boxed{P_i(x,t) = P_0\cos(\omega t - kx + \varphi_i)}$$ avec $k = \dfrac{2\pi}{c\cdot T} = \dfrac{2\pi}{\lambda}$.

3°] $P_i(L,t) = P_0\cos(\omega t - kL + \varphi_i)$. $\quad P_r(L,t) = P_0\cos(\omega t - kL + \varphi_i + \varphi_r)$.

4°] $P_r(x,t) = P_r(L, t-\tau')$ $\quad \tau' = \dfrac{L-x}{c}$.

$P_r(x,t) = P_0\cos\omega\left(t - \left(\dfrac{L-x}{c}\right)\right) - kL + \varphi_i + \varphi_r) = \boxed{P_0\cos(\omega t + kx - 2kL + \varphi_i + \varphi_r) = P_r(x,t)}$

5°] $P_{tot}(x,t) = P_0\cos(\omega t - kx + \varphi_i) + P_0\cos(\omega t + kx - 2kL + \varphi_i + \varphi_r)$

$= \underbrace{2P_0\cos\left(kx - kL + \dfrac{\varphi_r}{2}\right)}_{\text{Amplitude dépend de } x \,=\, A(x)}\cos\left(\omega t - kL + \varphi_i + \dfrac{\varphi_r}{2}\right)$. Onde stationnaire

6°] Ventre $P_{tot}(x)$ vibre avec une amplitude maximale.

$\cos\left(kx - kL + \dfrac{\varphi_r}{2}\right) = \pm 1$

$kx_N - kL + \dfrac{\varphi_r}{2} = n\pi$

$k(x_{N+1}) - kL + \dfrac{\varphi_r}{2} = (n+1)\pi$

$\Rightarrow k[x_{N+1} - x_N] = \pi$

$\dfrac{2\pi}{\lambda}\Delta x = \pi \rightarrow \boxed{\Delta x = \dfrac{\lambda}{2}}$

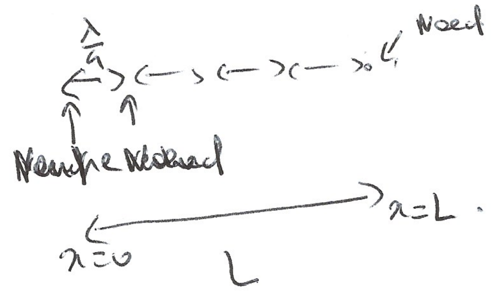
Noeud $P_{tot}(x) = 0 \rightarrow \cos\left(kx - kL + \dfrac{\varphi_r}{2}\right) = 0$

$k x_n - kL + \frac{\varphi_r}{2} = (2n+1)\frac{\pi}{2}$

$k x_{n+1} - kL + \frac{\varphi_r}{2} = (2(n+1)+1)\frac{\pi}{2}$

$\Rightarrow k\,\Delta x = \pi \Rightarrow \boxed{\Delta x = \frac{\lambda}{2}}$

7



$L = \frac{\lambda}{4} + n\frac{\lambda}{2} = (2n+1)\frac{\lambda_n}{4}$

$\lambda_n = \frac{4L}{2n+1} = \frac{c}{f_n'}$

$\Rightarrow f_n' = \underbrace{(2n+1)}_{n'}\frac{c}{4L}$

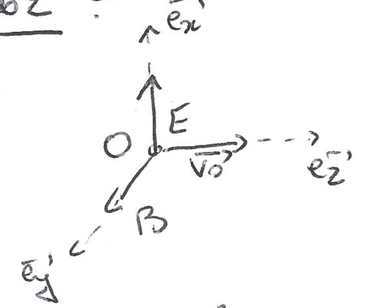
Noeud $P_{tot} = 0 \to \varphi_r = \pi$
en $x = L$

$f_n' = n'\frac{c}{4L}$ et n' impair.

$f_1 = \frac{c}{4L} = \frac{340}{4\cdot 1{,}4} = \frac{340}{5{,}6} = 60{,}7\ \mathrm{Hz} = 61\ \mathrm{Hz}$.

$\boxed{25D}$ $\boxed{26C}$ $\boxed{27D}$ $\boxed{28B}$ $\boxed{29C}$ $\boxed{30A}$.

Ex2



25D $B = 0$ $m\vec a = q\vec E = -e\vec E \to \boxed{\vec a = -\frac{e}{m}\vec E}$

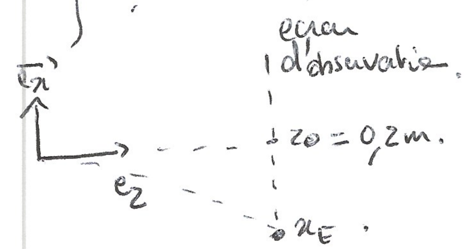
26 proj/PFD

proj/$\vec{e_x}$ $\ddot x = -\frac{e}{m}E \to \dot x = -\frac{e}{m}Et$

proj/$\vec{e_z}$ $\ddot z = 0 \to \dot z = v_0$

$\Leftarrow \overrightarrow{OM} = x\vec{e_x} + z\vec{e_z}$

27



$x = -\frac{e}{2m}Et^2$

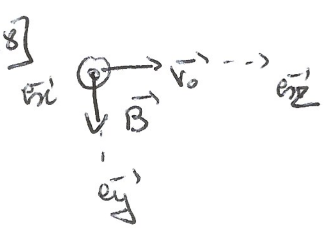
$z = v_0 t$

$\Rightarrow \boxed{x = -\frac{e}{2m}E\frac{z^2}{v_0^2}}$ $\to \boxed{26C}$

$x_E = -\frac{e}{2m}E\frac{z_0^2}{v_0^2}$ $e = 1{,}6\cdot10^{-19}\,\mathrm{C}$ $v_0 = 10^6\,\mathrm{m\,s^{-1}}$ $m = 10^{-30}\,\mathrm{kg}$ $E = 10\,\mathrm{V\,m^{-1}}$

$\Rightarrow \boxed{x_E = -4\,\mathrm{cm}}$ $\boxed{27D}$

28]



$\vec F_m = -e\vec{v_0}\wedge\vec B = -ev_0B\underbrace{\vec{e_z}\wedge\vec{e_y}}_{-\vec{e_x}} = +ev_0B\vec{e_x}$

$\boxed{28B}$

29] Trajectoire circulaire et uniforme (TEC) $\Rightarrow m\frac{d\vec v}{dt} = -e\vec v\wedge\vec B$ $\Rightarrow$ en coordonnées polaires $m\frac{v_0^2}{R} = ev_0B$

$\Rightarrow \boxed{R = \frac{mv_0}{eB}}$ 29C (2)

30] Mvt rectiligne uniforme $-e(\vec E + \vec{v_0}\wedge\vec B) = 0$ $\boxed{v_0 = \frac{E}{B}}$ 30A

Correction Pb 1.

II.1 a] $f_1 = 80440\ kHz > 20\ kHz$, $f_2 = 80000\ kHz > 20\ kHz$ } f_1 et f_2 ne sont pas audibles.

b] $s_1(t) = s_0 \sin 2\pi f_1 t$

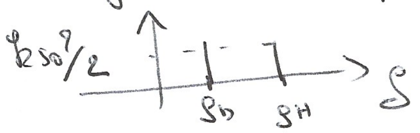
$s_2(t) = s_0 \sin 2\pi f_2 t$.

$$s(t) = k s_0^2 \sin 2\pi f_1 t \cdot \sin 2\pi f_2 t = \frac{k s_0^2}{2}\left[\cos 2\pi (f_1 - f_2)t - \cos 2\pi (f_1 + f_2)t\right].$$

$s(t)$ comporte 2 fréquences

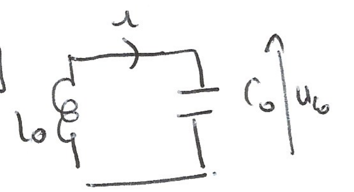
$f_b = f_1 - f_2 = 440\ Hz$ audible

$f_H = f_1 + f_2 = 160440\ Hz$ inaudible.



2] On veut garder uniquement $\frac{k s_0^2}{2} \cos 2\pi (f_1 - f_2)t$ donc éliminer la partie du signal à la fréquence f_H -> filtre passe-bas.

3] a]



loi des mailles + $\begin{cases} L_0 \frac{di}{dt} + u_{C_0} = 0. \\ i = C_0 \frac{du_{C_0}}{dt}. \end{cases}$

d'où $L_0 C_0 \frac{d^2 u_{C_0}}{dt^2} + u_{C_0} = 0$ oscillateur harmonique $\frac{d^2 u_{C_0}}{dt^2} + \omega_0^2 u_{C_0} = 0$.

c] => $\omega_0 = \frac{1}{\sqrt{L_0 C_0}}$ => $\boxed{f_2 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{L_0 C_0}}}$

b) $\boxed{u_{C_0}(t) = A \cos(\omega_0 t + \varphi)}$

d] Tension aux bornes de C continue =>, à $t = 0^+$ $u_C(t_0) = u_0$.

Courant traversant la bobine continu => $i(0^+) = i(0^-) = 0$

=> $u_0 = A \cos\varphi$

$0 = -\omega_0 A \sin\varphi \rightarrow \varphi = 0$

=> $u_0 = A$ $\boxed{u_{C_0}(t) = u_0 \cos(\omega_0 t)}$

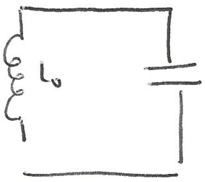
fig3°: oscillateur → signal à la fréquence f_1

Oscillateur local → " " f_2

Mélangeur → Multiplieur.

Sortie du filtre (avant l'ampli) →. Signal audible à la fréquence. $f_b = f_1 - f_2$.

5°] a]



$C_0 // C_{h_1} = C_0 + C_{h_1}$.

$$\boxed{f_1 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{L_0(C_0+C_{h_1})}}}$$

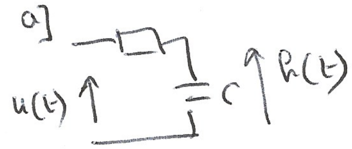
b] $f_b = f_2 - f_1 = \dfrac{1}{2\pi\sqrt{L_0C_0}} - \dfrac{1}{2\pi\sqrt{L_0(C_0+C_{h_1})}}$. → fréquence à garder.

$$f_H = f_2 + f_1 = \frac{1}{2\pi\sqrt{L_0C_0}} + \frac{1}{2\pi\sqrt{L_0(C_0+C_{h_1})}}$$

$f_c > 20$ kHz pour garder les fréquences audibles.
< 160 kHz pour éliminer f_H.

<u>ex</u> $f_c =$.

6°] a]



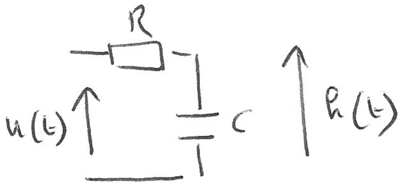
$T(jf) = \dfrac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + R} = \dfrac{1}{1+jRC\omega}$ Filtre passe bas 1° ordre.

$|T|(f_c) = \dfrac{|T|_{max}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \omega_c = \dfrac{1}{RC}$. et $\boxed{f_c = \dfrac{1}{2\pi RC}}$.

<u>ex</u> $f_c = 8$ kHz

b] $C = 0{,}01\,\mu F = 10^{-8}$ F. $\Rightarrow R = \dfrac{1}{2\pi f_c \cdot C} = 1\,k\Omega$.

C.



$$\underline{H} = \frac{\frac{1}{jC\omega}}{R + \frac{1}{jC\omega}} = \frac{1}{1+jRC\omega} = \frac{1}{1+jx}$$

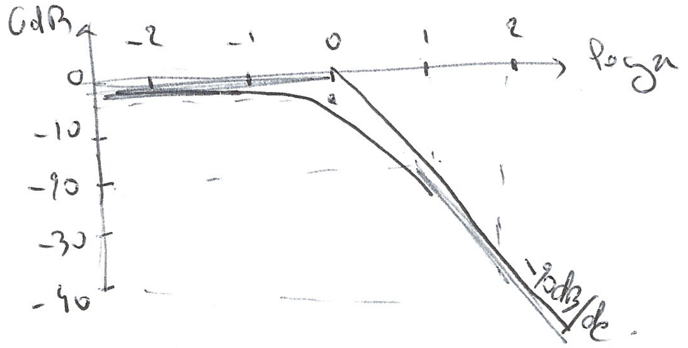
$x = \frac{\omega}{\omega_0}$ $\underline{H} \rightarrow 1$ qd $x < 0,1$ $\Rightarrow$ $G_{dB} \rightarrow 0 = Y_{BH}$

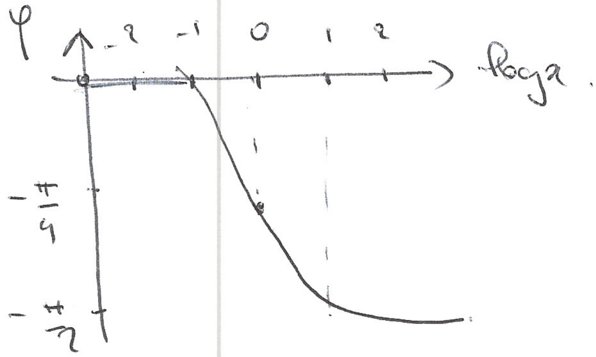
$\varphi \rightarrow 0$

$\underline{H} \rightarrow \frac{1}{jx}$ quand $x > 10$ $\rightarrow$ $G_{dB} \rightarrow -20 \log x = Y_{HF}$.

$\varphi \rightarrow -\frac{\pi}{2}$

$Y_{BH} = Y_{HF}$ pour $x = 1$. $\varphi = -\frac{\pi}{4}$ pour $x = 1$. $G_{dB} \rightarrow -3dB$ pour $x = 1$





7] la₃ $f = 440$ Hz $\rightarrow x = 20$ cm.

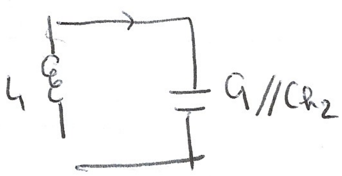
8] $x \nearrow f \searrow$ ~~[illegible]~~ + ~~grave~~. son + grave $x \searrow$ main rapprochée.

$f_2 = 2f_1 \rightarrow$ 1 octave $\rightarrow$ instrument couvre 5 octaves.

Pour doubler la fréquence, la main se déplace de 12 cm.

II.2.

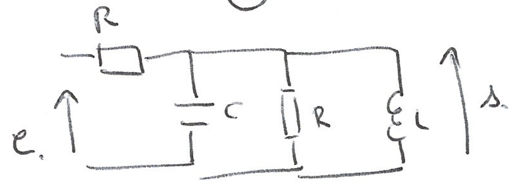
9] Circuit oscillant. $L_1, C_1 // C_{h2}$. + filtre passe bande + détecteur.



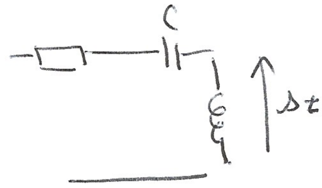
Voir question précédente.

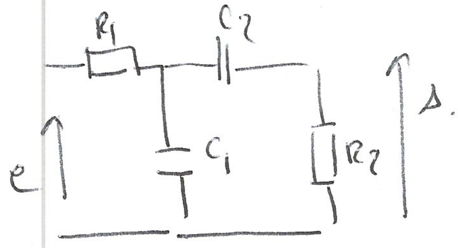
$$\boxed{f' = \frac{1}{2\pi\sqrt{L_0(C_0+C_{h2})}}}$$

(a)

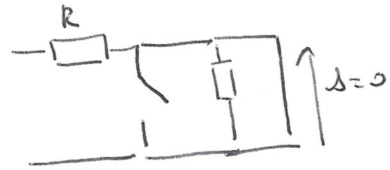


(a)

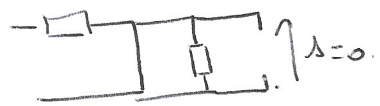




BF



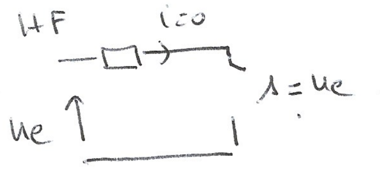
HF.



Passe bande

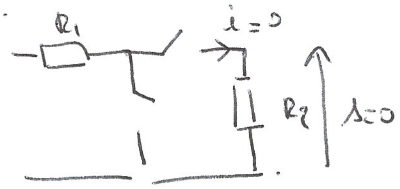
BF

HF

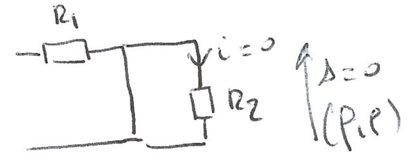


Passe haut.

BF.



HF

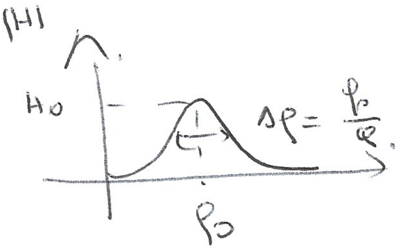


Passe bande.

1) $H_0 = |H(f=0)| = H_{max}$.

$f_0 =$ fréquence de résonance

$Q = \frac{f_0}{\Delta f} \rightarrow \Delta f = \frac{f_0}{Q}.$



2) $$\underline{H} = \frac{Z_{eq}}{R+Z_{eq}} = \frac{1}{1+RY_{eq}} = \frac{1}{1+R\left[\frac{1}{R}+jC\omega+\frac{1}{jL\omega}\right]} = \frac{1}{2+\left(jRC\omega+\frac{R}{jL\omega}\right)}$$

$$= \frac{1/2}{1+j\left(\frac{RC\omega}{2}-\frac{R}{2L\omega}\right)} = \frac{H_0}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}.$$

$\boxed{H_0 = \frac{1}{2}}$.

$$\begin{cases} \frac{RC}{2} = \frac{Q}{\omega_0} \\ \frac{R}{2L} = Q\,\omega_0 \end{cases} \qquad Q^2 = \frac{R^2 C}{4L} \rightarrow \boxed{Q = \frac{R}{2}\sqrt{\frac{C}{L}}}.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}.$$

$$U_m = V_m \cdot \frac{H_0}{\sqrt{1+Q^2\left(\frac{f}{f_0}-\frac{f_0}{f}\right)^2}}.$$

Correction Pb2 -

1] Mvt circulaire $m\frac{d\vec{v}}{dt} = \ominus \frac{G m M_S}{r^2}\vec{e_r}$ et $\frac{d\vec{v}}{dt} = r\ddot{\theta}\vec{e_\theta} - r\dot{\theta}^2\vec{e_r}$

d'où $\boxed{\dot{\theta} = \sqrt{\frac{G M_S}{r^3}}}$ et $= T = \frac{2\pi}{\dot{\theta}}$ (car $\theta = \dot{\theta} t$).

d'où $T^2\dot{\theta}^2 = 4\pi^2 \Rightarrow \boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{G M_S}}$

Pour une ellipse de demi grand axe a → Généralisation $\boxed{\frac{T^2}{a^3} = \frac{4\pi^2}{G M_S}}$

AN $a_c = \frac{33 \cdot 10^{11}}{2\pi}$ → $\boxed{T_c = 6{,}5 \text{ ans}}$

2] $\vec{\sigma_S} = \vec{SM} \wedge m\vec{v}$ et $\frac{d\vec{\sigma_S}}{dt} = \underbrace{\vec{OM}\wedge\vec{F}}_{\text{colinéaire à } \vec{OM}} = 0$ → $\vec{\sigma_S} = \vec{cste}$ { en direction et en norme

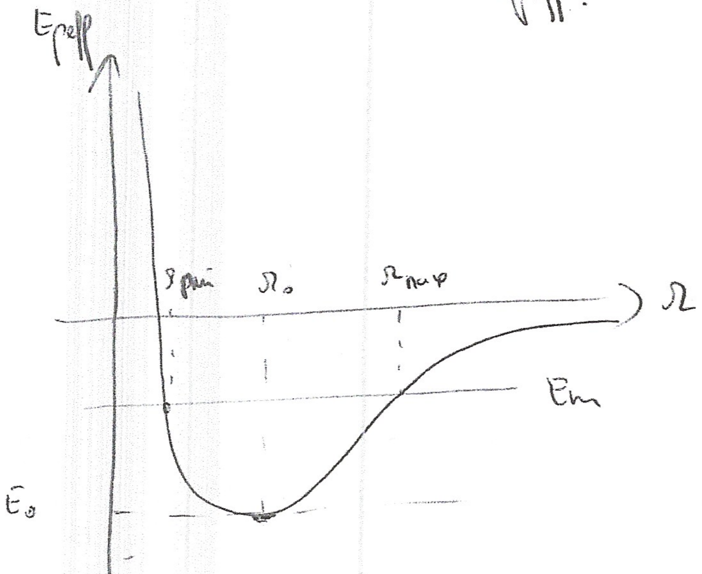
or $\vec{\sigma_S} = \vec{SM} \wedge m\vec{v} \Rightarrow \vec{\sigma_S} \perp$ plan $(\vec{SM}, \vec{v})$ qui garde donc la même direction puisque $\vec{\sigma_S}$ est de direction cste.

$C = r^2\dot{\theta}$

i] $E_m = \frac{1}{2} m v^2 - \mathcal{G}\frac{m M_S}{r}$ et $v^2 = \dot{r}^2 + r^2\dot{\theta}^2$

$= \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \mathcal{G}\frac{m M_S}{r}$ avec $\dot{\theta} = \frac{C}{r^2}$

d'où $E_m = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{1}{2} m \frac{C^2}{r^2} - \mathcal{G}\frac{m M_S}{r}}_{E_{p,eff}}$.



(Tracé → voir cours)

$\frac{1}{2} m \dot{r}^2 > 0 \Rightarrow E_m > E_{peff}$

Sur le graphique de l'énoncé $r_{max} = a + ae = a(1+e)$

$r_{min} = a - ae = a(1-e)$

j] La trajectoire circulaire correspond au minimum de l'E_{peff}.

$\frac{dE_{peff}}{dr} = -\frac{mC^2}{r^3} + \mathcal{G}\frac{m M_S}{r^2} = 0$ pour $r = r_0$ d'où $\boxed{r_0 = \frac{C^2}{\mathcal{G} M_S}}$

et $E_0 = E_{peff}(r = r_0)$ ou

$E_c(r_0) = \frac{1}{2} m (r\dot{\theta})^2 = \frac{1}{2} m r_0^2 \cdot \frac{C^2}{r_0^4} = \frac{1}{2} m \frac{C^2}{r_0^2} = \cancel{\frac{1}{2}\mathcal{G}\frac{m M_S}{C^2}} = \frac{1}{2} m \frac{r_0 \cdot \mathcal{G} M_S}{r_0^2} = \frac{1}{2}\frac{\mathcal{G} m M_S}{r_0}$

et $E_p(r_0) = -\mathcal{G}\frac{m M_S}{r_0}$ d'où $E_0 = E_c(r_0) + E_p(r_0) = \boxed{-\mathcal{G}\frac{m M_S}{2 r_0} = E_0}$

j]. $E_m = \frac{1}{2} m \dot{r}^2 + \frac{mC^2}{2r^2} - \mathcal{G}\frac{m M_S}{r}$ en A et P $\dot{r} = 0$ donc r_A et r_P solutions

de $E_m = \frac{mC^2}{2r^2} - \mathcal{G}\frac{m M_S}{r}$ $\Rightarrow$ $\boxed{E_m \cdot r^2 + \mathcal{G} m M_S r - \frac{1}{2} m C^2 = 0.}$

$$\Delta = (\mathcal{G}\, m\, M_S)^2 + 2E_m \cdot m C^2 > 0 \quad \text{si} \quad E_m > -\frac{1}{2m}\left(\frac{\mathcal{G}\, m M_S}{C}\right)^2$$

or $r_0 = \dfrac{C^2}{\mathcal{G} M_S}$ $\rightarrow$ $E_m > \underbrace{-\dfrac{1}{2}\dfrac{\mathcal{G}\, m M_S}{r_0}}_{E_0}$

Somme des racines

$\Rightarrow$ $r_{min} + r_{max} = -\dfrac{\mathcal{G}\, m M_S}{E_m} = 2a$ $\rightarrow$ $\boxed{E_m = -\dfrac{\mathcal{G}\, m M_S}{2a}}$

de plus $r_{min} \cdot r_{max} = a^2(1-e^2) = -\dfrac{m C^2}{2 E_m}$ (produit des racines)

$\rightarrow$ $\boxed{(1-e^2) = -\dfrac{m C^2}{2a^2 E_m}}$

6°] Vitesse aréolaire $\dfrac{dA}{dt} = \dfrac{|C|}{2} = \text{cte}$ Kepler.

Sur une période de l'ellipse $A = \dfrac{|C|}{2} \cdot T$

$\pi a^2 \sqrt{1-e^2} = \dfrac{|C|}{2} \cdot T$

en élevant au carré $\pi^2 a^4 (1-e^2) = \dfrac{C^2}{4} \cdot T^2$

$$-\pi^2 a^4 \frac{m \cancel{C^2}}{2a^2 E_m} = \frac{\cancel{C^2}}{4} T^2$$

$$\pi^2 a^4 \frac{m}{\cancel{2}a^2 \dfrac{\mathcal{G}\, m M_S}{\cancel{2}a}} = \frac{T^2}{4}$$

On retrouve $\boxed{\dfrac{T^2}{a^3} = \dfrac{4\pi^2}{\mathcal{G} M_S}}$