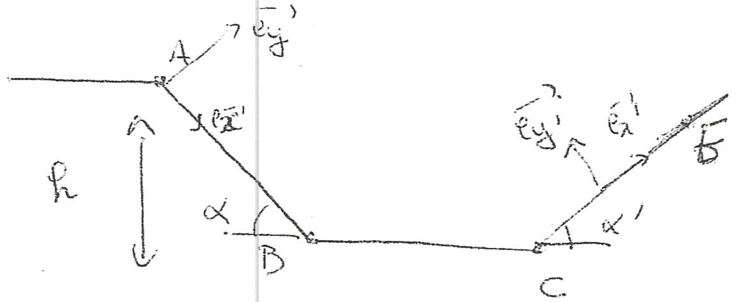
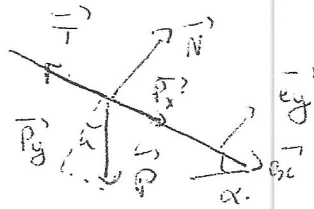
Correction DS4 24/01/25.

Cours

Application.



a) TEC $\quad \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = W_{A\to B}(\vec{P}) + \underbrace{W_{A\to B}(\vec{N})}_{0 \text{ car } \vec{N} \perp \vec{dl}} + W_{A\to B}(\vec{T})$.



$W_{A\to B}(\vec{P}) = + mgh$.

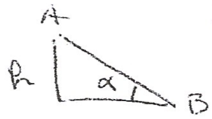
Calcul de $\vec{N}$ et $\vec{T}$ avec $\|\vec{T}\| = f \|\vec{N}\|$.

$\vec{T} = -\|\vec{T}\| \vec{e_x} = -T \vec{e_x}$ et le PFD projeté suivant

$\vec{e_y}'$ donne $\quad \vec{P_y} + \vec{N} = \vec{0}$

$(-mg\cos\alpha + N)\,\vec{e_y}' = 0 \Rightarrow N = mg\cos\alpha$ et $T = f\cdot mg\cos\alpha$.

$W_{A\to B}(\vec{T}) = -T\cdot AB$



$\sin\alpha = \frac{h}{AB} \Rightarrow AB = \frac{h}{\sin\alpha}$.

d'où $\quad \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = mgh - f mgh \frac{\cos\alpha}{\sin\alpha} = mgh\left(1 - \frac{f}{\tan\alpha}\right)$.

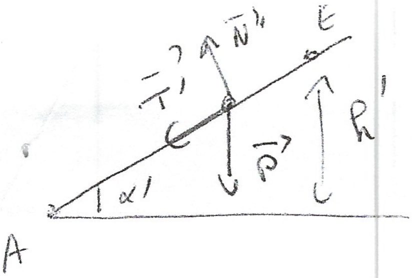
$\left(1 - \frac{f}{\tan\alpha}\right) = \frac{v_B^2 - v_A^2}{2\cdot g\cdot h} \Rightarrow \frac{f}{\tan\alpha} = 1 - \frac{(v_B^2 - v_A^2)}{2gh} \Rightarrow$

$$\boxed{\tan\alpha = \frac{f}{1 - \frac{(v_B^2 - v_A^2)}{2gh}}}$$

AN $\boxed{\alpha = 4{,}7°}$

2°) $\frac{1}{2} m v_E^2 - \frac{1}{2} m v_C^2 = W_{C\to E}(\vec{P}) + \underbrace{W_{C\to E}(\vec{N}')}_{0} + W_{C\to E}(\vec{T}')$.

De la même façon : $\|\vec{T}'\| = f\|\vec{N}'\| = f\cdot mg\cos\alpha'$



$AE = D$ et $v_E = 0$

$W_{C\to D}(\vec{P}) = -mgh'$ et $\sin\alpha' = \frac{h'}{D}$

$= -mgD\cdot\sin\alpha'$

$W_{C\to D}(\vec{T'}) = \ominus f mg\cos\alpha' \cdot D.$

d'où $-\frac{1}{2}\cancel{m}v_B^2 = -\cancel{m}gD\sin\alpha' - f\cancel{m}g\cos\alpha'\cdot D.$

$\frac{v_B^2}{2} = g[\sin\alpha' + f\cos\alpha']\cdot D \Rightarrow \boxed{D = \frac{v_B^2}{2g(\sin\alpha' + f\cos\alpha')}}.$

AN

$\alpha' = \alpha = 4{,}7°$ $D = 4{,}4\,m.$

$\alpha' = 10°$ $D = 2{,}2\,m.$

b] Le colis reste immobile en E si $\|\vec{T}\| < f\cdot\|\vec{N}\|.$

avec $\|\vec{N'}\| = mg\cos\alpha'$ $\Rightarrow$ $mg\sin\alpha' < f\cdot mg\cos\alpha$

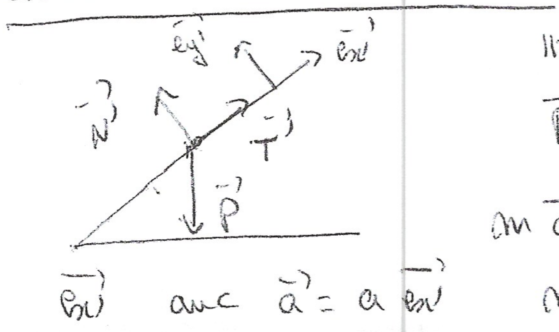
$\underbrace{\|\vec{T}\| = mg\sin\alpha'}_{\text{projection de ~~PFD~~ relation } \Sigma\vec{F}=0 \text{ suivant } \vec{e_x'}}$ $\dashrightarrow \boxed{\tan\alpha' < f.}$

AN $\alpha' = 4{,}7°.$ $\tan\alpha' = 0{,}08$ $f = 0{,}1$ Condition vérifiée.

$\alpha' = 10°$ $\tan\alpha' = 0{,}17 > f \Rightarrow$ Condition non vérifiée.

$\rightarrow$ le colis redescend.

Etude de la descente.



$\|\vec{T'}\| = f\cdot mg\cos\alpha'$ et $\vec{T'} = +fmg\cos\alpha'\,\vec{e_x'}$

$\vec{P'} = -mg\sin\alpha'\,\vec{e_x'} - mg\cos\alpha'\,\vec{e_y'}.$

$m\vec{a'} = \vec{P'} + \vec{T'} + \vec{N}$ donc en projection suivant $\vec{e_x'}$ avec $\vec{a'} = a\,\vec{e_x'}$

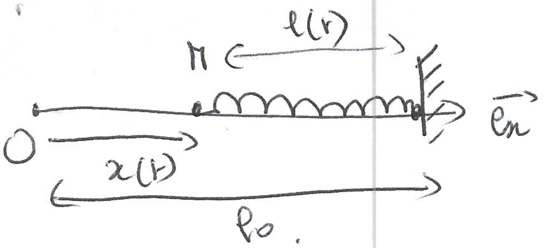
$ma = -mg\sin\alpha' + fmg\cos\alpha' \Rightarrow$

$\boxed{a = -g\cos\alpha'[\tan\alpha' - f] < 0.}$

$a = -0{,}75\,m\,s^{-2}$

2

Ex 2 Modélisation d'un haut-parleur.



$\vec{OM} = x(t)\vec{e_x} \rightarrow \vec{v} = \dot{x}\vec{e_x} \rightarrow \vec{a} = \ddot{x}\vec{e_x}$

$\vec{P} = -mg\vec{e_y}$ $\vec{R} = R\vec{e_y}$

$\vec{F} = Ki(t)\vec{e_x}$ $\vec{F}(t) = -\delta\vec{v}(t) = -\delta\dot{x}\vec{e_x}$

$\vec{T} = -k(\ell-\ell_0)(-\vec{e_x})$ et $\ell = \ell_0 - x(t)$.

$\vec{T} = -k(\cancel{\ell_0} - x(t) - \cancel{\ell_0})(-\vec{e_x})$

$= -kx(t)\vec{e_x}$.

1°] équation différentielle

$m\vec{a} = \Sigma\vec{F} \rightarrow$ projeté suivant $\vec{e_x}$: on obtient.

$m\ddot{x} = Ki(t) - \delta\dot{x} - kx \Rightarrow m\ddot{x} + \delta\dot{x} + kx = KI_m\cos\omega t$.

$$\boxed{\ddot{x} + \frac{\delta}{m}x + \frac{k}{m} = \frac{KI_m}{m}\cos\omega t} \Rightarrow \begin{cases} \frac{\delta}{m} = \frac{\omega_0}{Q} \\ \frac{k}{m} = \omega_0^2 \rightarrow \boxed{\omega_0 = \sqrt{\frac{k}{m}}} \end{cases}$$

d'où $Q = \frac{m\omega_0}{\delta} = \boxed{\frac{\sqrt{km}}{\delta} = Q}$.

AN $k = 15\cdot10^3\,\mathrm{N\,m^{-1}}$ $m = 10\,g = 10\cdot10^{-3}\,kg = 10^{-2}\,k \rightarrow \boxed{\omega_0 = 1225\,\mathrm{rad\,s^{-1}}}$

2°] On cherche $x(t) = X_m\cos(\omega t+\varphi) \rightarrow \underline{x}(t) = \underline{X_m}e^{j\varphi}$ avec $\underline{X_m} = X_m e^{j\varphi}$

$A \leftarrow \frac{KI_m}{m}\cos\omega t \rightarrow Ae^{j\omega t} = \underline{A}(t)$

$\underline{A}(t)$ et $\underline{X_m}(t)$ vérifient la n eq. diff $\Rightarrow \left((j\omega)^2 + j\omega\cdot\frac{\omega_0}{Q} + \omega_0^2\right)\underline{X_m} = A$

$$\frac{\underline{X_m}}{A} = \frac{1}{\omega_0^2 + j\omega\cdot\frac{\omega_0}{Q} + (j\omega)^2} \Rightarrow \frac{\frac{1}{\omega_0^2}}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2} = \frac{mX_m e^{j\varphi}}{KI_m}.$$

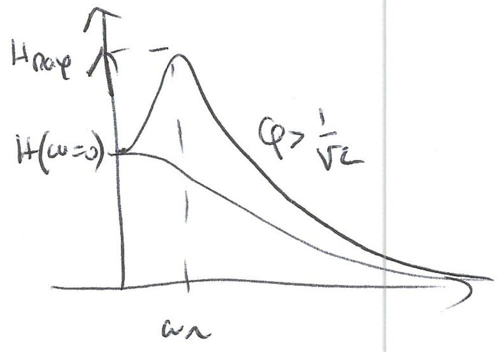
$$\Rightarrow \underline{H}(j\omega) = \frac{X_m e^{j\varphi}}{KI_m} = \frac{\frac{1}{m\omega_0^2}}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2} = \boxed{\frac{1/k}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2} = \underline{H}}$$

Rq $[|\underline{H}|] = \frac{L}{[\text{Force}]}$ et $\left[\frac{1}{k}\right] = \frac{L}{N}$.

Filtre passe bas second ordre.

$\varphi > \frac{1}{\sqrt{2}}$ Filtre résonant $H_{max} = H(\omega=0)\cdot\frac{\varphi}{\sqrt{1-\frac{1}{4\varphi^2}}}$ pour $\omega_r = \omega_0\sqrt{1-\frac{1}{2\varphi^2}}$

$\varphi \leq \frac{1}{\sqrt{2}}$ Filtre non résonant.



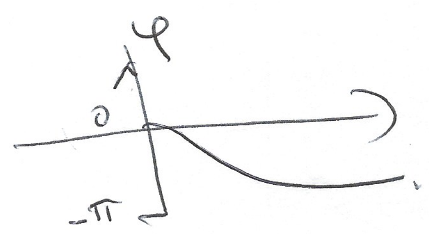
5°] $\underline{H} = \frac{\frac{1}{k}}{1-x^2+j\frac{x}{\varphi}}$

$x<1$ $1-x^2>0 \Rightarrow \varphi = -\arctan\frac{x}{\varphi(1-x^2)}$ $\begin{cases} x \to 0 \to \varphi = 0 \\ x \to 1^- \end{cases}$

$\varphi \to -\arctan\left(\frac{1}{0^+}\right) \xrightarrow{\quad} = -\frac{\pi}{2}$

$x>1$ $(1-x^2)<0$ $\frac{x}{\varphi}>0$ $\Rightarrow \varphi = -\left[\arctan\frac{x}{\varphi(1-x^2)} + \pi\right]$

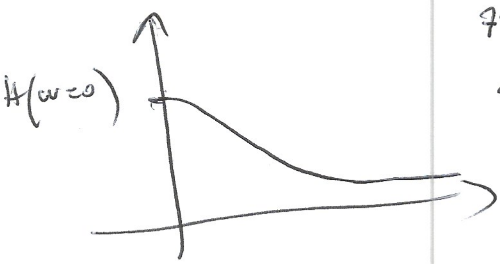
$x \to 1^+$ $\varphi \to -\left[\underbrace{\arctan\frac{1}{0^-}}_{-\frac{\pi}{2}} + \pi\right] = -\frac{\pi}{2}$ $\frac{1}{0^-} \to -\infty$



$x \to \infty$ $\varphi \to -\left[\arctan\frac{x}{-\varphi x^2} + \pi\right] = -\pi$ $\frac{x}{-\varphi x^2} \to -\frac{1}{\varphi x} \to 0$

0

6°] $\varphi = \frac{1}{\sqrt{2}} = \frac{\sqrt{km}}{f} \to f = \frac{\sqrt{km}}{\varphi} = \sqrt{2km} \Rightarrow \boxed{f = 17{,}3\ kg\,s^{-1}}$



7°) $\omega = 6280\ rd\,s^{-1}$.

$X_m = \frac{KI_m/k}{\sqrt{(1-x^2)^2 + \frac{x^2}{\varphi^2}}}$ avec $x = \frac{\omega}{\omega_0} = \frac{6280}{1225}$.

$\varphi = \frac{1}{\sqrt{2}}$

$\varphi = -\left[\arctan\frac{x}{\varphi(1-x^2)} + \pi\right]$ $\Rightarrow \begin{cases} X_m = 0{,}5\ mm. \\ \varphi = -2{,}86\ rd \end{cases}$

8°] Calcul de la bande passante.

$Q = \frac{1}{\sqrt{2}}$; le filtre est non résonant.

$$H = \frac{H(\omega=0)}{1 + j\frac{x}{Q} + (jx)^2} = \frac{H(\omega=0)}{1 - x^2 + j\sqrt{2}\cdot x}.$$ et $H(\omega=0) = H_{max}$.

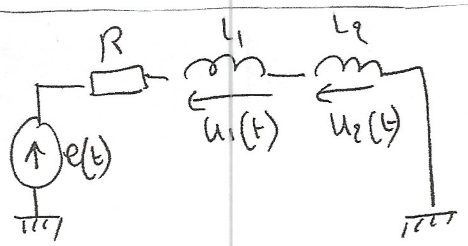
$$\underline{H}(\omega_c) = \frac{H_{max}}{\sqrt{2}} \Rightarrow \frac{H_{max}}{\sqrt{(1-x^2)^2 + 2x^2}} = \frac{H_{max}}{\sqrt{2}}$$

$(1-x^2)^2 + 2x^2 = 2$

$1 - \cancel{2x^2} + x^4 + \cancel{2x^2} = 2$ $\quad 1 + x^4 = 2 \rightarrow x^4 = 1 \Rightarrow \omega_c = \omega_0$.

Bande passante $\Delta\omega = [0 ; \omega_0] = 1825 \text{ rad s}^{-1}$.

Ex3.



1) $\underline{u_1} = \dfrac{jL_1\omega \cdot E}{R+j(L_1+L_2)\omega}$ $\qquad \underline{u_2} = \dfrac{jL_2\omega \cdot E}{R+j(L_1+L_2)\omega}$

2) $\underline{u_S} = \underline{u_1} - \underline{u_2} = \dfrac{j(L_1-L_2)\omega \cdot E}{R+j(L_1+L_2)\omega} = \boxed{\dfrac{j\left(\frac{L_1-L_2}{R}\right)\omega \cdot E}{1+j\left(\frac{L_1+L_2}{R}\right)\omega}} = \underline{H}$

A identifier à $\underline{H} = \dfrac{H_0 \cdot j\frac{\omega}{\omega_0}}{1+j\frac{\omega}{\omega_0}}$ $\to$ $\boxed{\omega_0 = \dfrac{R}{L_1+L_2}}$

$\dfrac{H_0}{\omega_0} = \dfrac{L_1 - L_2}{R}$ $\to$ $\boxed{H_0 = \dfrac{L_1-L_2}{L_1+L_2}}$

$L_1 = L_e\left(1+\dfrac{\Delta z}{\delta}\right)$
$L_2 = L_e\left(1-\dfrac{\Delta z}{\delta}\right)$

$\left\{ \begin{array}{l} L_1 + L_2 = 2L_e \\ L_1 - L_2 = 2L_e\dfrac{\Delta z}{\delta} \end{array} \right.$ $\Rightarrow$ $\boxed{\omega_0 = \dfrac{R}{2L_e}}$ $\boxed{H_0 = \dfrac{\Delta z}{\delta}}$

Rq. $\Delta z > 0$ ou $\Delta z < 0$

On pose $x = \dfrac{\omega}{\omega_0}$

$\underline{H} = \dfrac{H_0 \cdot jx}{1+jx}$

$\xrightarrow{BF}$ $H_0 \cdot jx \Rightarrow G_{dB} \to Y_{BF} = 20\log|H_0| + 20\log x$

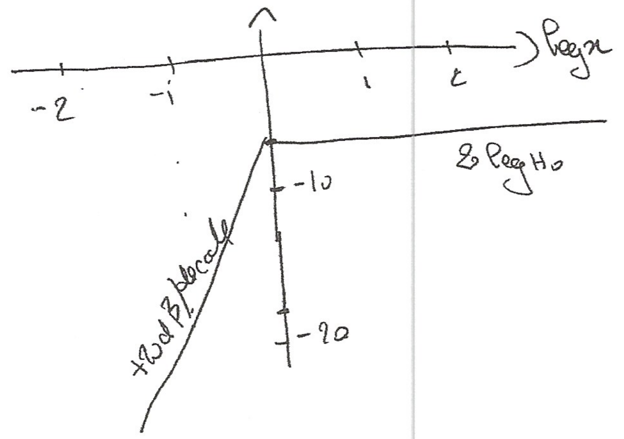
$\to$ $H_0 \to G_{dB} \to Y_{HF} = 20\log|H_0|$.

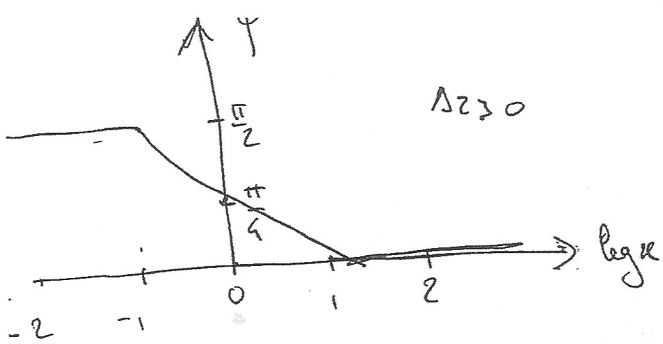
Y_{BF} et Y_{HF} se croisent en $x=1$; $Y = 20\log|H_0|$.

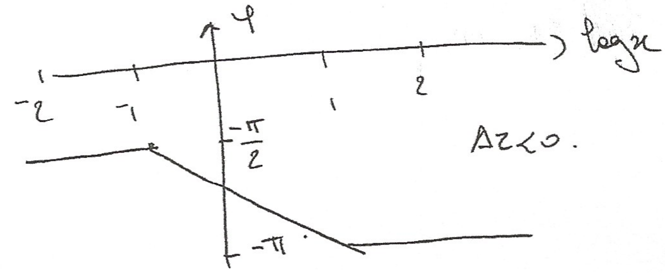
$\varphi(x) = \pm\dfrac{\pi}{2} - \arctan x$ ($+$: $\Delta z>0$; $-$: $\Delta z<0$)

$\underline{H} \xrightarrow{BF} H_0 jx$. $\qquad \Delta z > 0 \Rightarrow \varphi_{BF} \to \dfrac{\pi}{2}$
$\qquad \Delta z < 0 \to \varphi_{BF} \to -\dfrac{\pi}{2}$.

$\underline{H} \xrightarrow{HF} H_0$ $\qquad \Delta z > 0 \to \varphi_{HF} \to 0$.
$\qquad \Delta z < 0 \quad \varphi_{HF} \to -\pi$







Filtre passe haut 1° ordre.

$\omega_0 = \omega_c$ pulsation de coupure car.

$|H(\omega_c)| = \dfrac{H_{max}}{\sqrt{2}} = \dfrac{H_0}{\sqrt{2}}$

A haute fréquence $f \gg f_0$ donc $f > f_0$ $H \to H_0 = \dfrac{\Delta Z}{\delta}$.

$f_0 = \dfrac{\omega_0}{2\pi} = \dfrac{R}{2\pi \cdot 2L_e} = \dfrac{750}{2\pi \cdot 2 \cdot 60 \cdot 10^{-3}} = 9{,}9 \cdot 10^{2}\,Hz \simeq 10^{3}\,Hz.$

$f = 4\,kHz \simeq 4 f_0$

$s(t) = E \cdot |H(f)| \cos(\omega t + \varphi_H(f))$ On considère $|H(f)| \simeq |H_0|$ pour $f = 4\,kHz$

$\varphi = 0$ ou π suivant que $\Delta Z > 0$ ou $\Delta Z < 0$

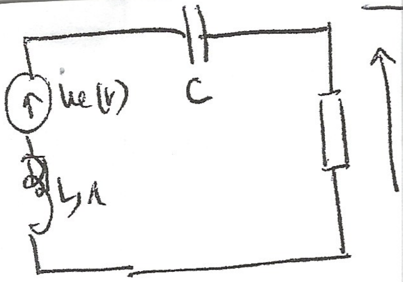
$s(t) = E \cdot \dfrac{|\Delta Z|}{\delta} \cos(\omega t + \varphi)$ si $\Delta Z > 0$ $|\Delta Z| = \Delta Z$ et $\varphi = 0$

si $\Delta Z < 0$ $|\Delta Z| = -\Delta Z$ et $\varphi = \pi$.

si $\boxed{\Delta Z > 0} \Rightarrow \boxed{U_s(t) = E \cdot \dfrac{\Delta Z}{\delta} \cos\omega t.}$

$\Delta Z < 0$ $U_s(t) = -E \dfrac{\Delta Z}{\delta} \cos(\omega t - \pi) = \boxed{E \cdot \dfrac{\Delta Z}{\delta} \cos\omega t.}$ $\Leftarrow \Delta Z < 0$

Log Rappel (cours) 1°) $u_s(t) = Ri(t)$



BF $\dashv\vdash \Leftrightarrow$ interrupteur ouvert $\Rightarrow i = 0 \Rightarrow u_s = 0 = Ri$

HF bobine $\Leftrightarrow$ interrupteur ouvert $\Rightarrow i = 0 \Rightarrow u_s = 0. = Ri$

Filtre passe-bande.

2°) $u_e(t) = E\cos\omega t \rightarrow \underline{u_e} = E$

$u_s(t) = U_s\cos(\omega t + \varphi) \Rightarrow \underline{u_s} = U_s e^{j\varphi}$.

3°) $\underline{H} = \dfrac{R}{R + r + jL\omega + \dfrac{1}{jC\omega}} = \boxed{\dfrac{R/(R+r)}{1 + j\left(\dfrac{L\omega}{R+r} - \dfrac{1}{(R+r)C\omega}\right)} = \underline{H}}$

4°) $\underline{H} = \dfrac{H_0}{1 + jQ\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)} \Rightarrow \begin{cases} H_0 = \dfrac{R}{R+r}. \\ \dfrac{Q}{\omega_0} = \dfrac{L}{R+r} \\ Q\omega_0 = \dfrac{1}{(R+r)C} \end{cases} \Rightarrow Q^2 = \dfrac{1}{R+r} \cdot \dfrac{L}{C}$

$\Rightarrow \boxed{Q = \dfrac{1}{R+r}\sqrt{\dfrac{L}{C}}}$ et $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}.}$

5°] $H(\omega) = |\underline{H}| = \dfrac{H_0}{\sqrt{1 + Q^2\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2}}$ max pour $\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega} = 0 \rightarrow \boxed{\omega_r = \omega_0}$

$H_0 = H(\omega_r) = \max$.

6°) $H(\omega_1) = H(\omega_2) = \dfrac{H_0}{\sqrt{2}}$; en posant $x = \dfrac{\omega}{\omega_0}$, on obtient.

$\dfrac{H_0}{\sqrt{1 + Q^2\left(x - \dfrac{1}{x}\right)^2}} = \dfrac{H_0}{\sqrt{2}} \rightarrow x - \dfrac{1}{x} = \pm\dfrac{1}{Q} \rightarrow x^2 \pm \dfrac{x}{Q} - 1 = 0.$

$x^2 + \frac{x}{Q} - 1 = 0$ $\qquad$ $x^2 - \frac{x}{Q} - 1 = 0$

$$\Delta = \frac{1}{Q^2} + 4.$$

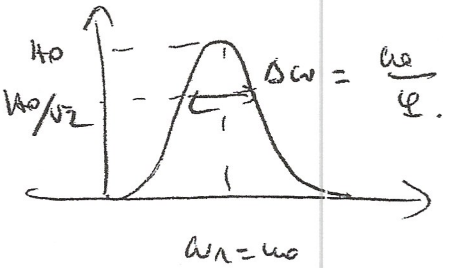
$x = -\frac{1}{2Q} \pm \sqrt{\frac{\Delta}{2}}$ $\qquad$ $x = \frac{1}{2Q} \pm \sqrt{\frac{\Delta}{2}}$.

En ne gardant que les racines positives $\quad x_1 = -\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}$

$$x_2 = \frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}.$$

d'où $x_2 - x_1 = \frac{1}{Q} = \frac{\Delta\omega}{\omega_0} \Rightarrow \boxed{Q = \frac{\omega_0}{\Delta\omega}}$

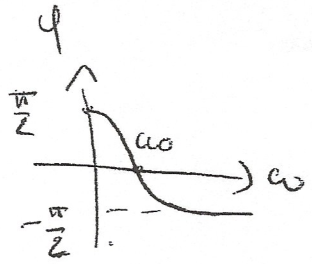
7°)



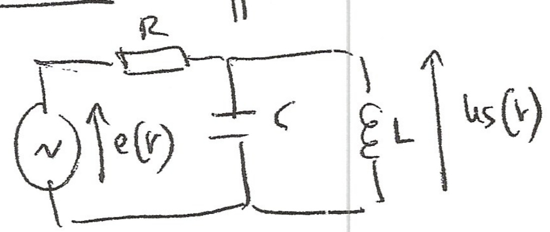
8°) $\varphi = -\arctan Q\left(x - \frac{1}{x}\right)$.

$\omega = \omega_r \to x = 1 \to \varphi = 0.$

$\left.\begin{matrix}\omega = \omega_1 \\ \text{et} \\ \omega_2\end{matrix}\right\}$ définis par $x - \frac{1}{x} = \pm\frac{1}{Q} \to \varphi = \pm\frac{\pi}{4}$.



Partie 2 Application.



$e(t) = E\cos\omega t \to \underline{e} = E$

$u_s(t) = U_s\cos(\omega t + \varphi) \to \underline{u_s} = U_s.$

BF — bobine $\Leftrightarrow$ — $\to u_s = 0$ \
HF — condensateur $\Leftrightarrow$ — $\to u_s = 0$ \} Filtre passe bande.

3°) $\underline{A}(j\omega) = \frac{Z_{eq}}{R + Z_{eq}} = \frac{1}{1 + RY_{eq}} = \frac{1}{1 + R\left(jC\omega + \frac{1}{jL\omega}\right)} = \frac{1}{1 + jR\left(C\omega - \frac{1}{L\omega}\right)}$

$$\begin{cases} H_0 = 1 \\ \dfrac{Q}{\omega_0} = RC \\ Q\omega_0 = \dfrac{R}{L} \end{cases} \rightarrow Q^2 = R^2 \cdot \frac{C}{L} \rightarrow \boxed{Q = R\sqrt{\frac{C}{L}}}$$

et $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$

5°] $\omega_0 = \omega_r \Rightarrow$ pulsation pour laquelle $H(\omega)$ et donc $U_s(\omega)$ est maximal.

$\omega_0 = \omega_r \Rightarrow \varphi(\omega_0) = 0 \Rightarrow$ d'où $\boxed{\omega_0 = 10^4 \, rad\,s^{-1}}$

$$\left.\begin{array}{l} \omega_1 = 7 \cdot 10^3 = 7000 \, rad\,s^{-1} \\ \omega_2 = 1{,}6 \cdot 10^4 = 16000 \, rad\,s^{-1} \end{array}\right\} \Delta\omega = 9000 \, rad\,s^{-1}$$

$$\rightarrow Q = \frac{\omega_0}{\Delta\omega} = \boxed{1 = Q.}$$