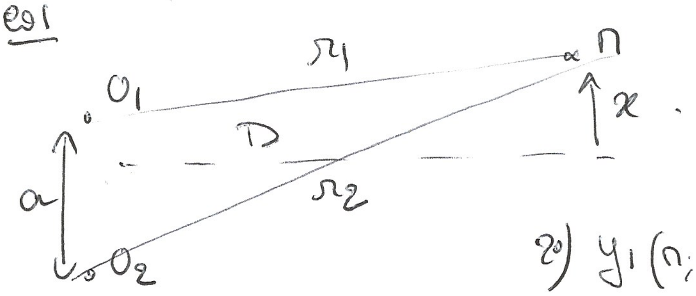
Correction TD interférences - ondes stationnaires.

ex1



1°) $y_1(O_1;t) = s_0 \cos \omega t$ $\qquad \omega = \frac{2\pi}{T}$.

$y_2(O_2;t) = s_0 \cos \omega t$.

2°) $y_1(M;t) = y_1(O_1; t - \frac{r_1}{c})$

$y_2(M;t) = y_2(O_2; t - \frac{r_2}{c})$.

$$\begin{cases} y_1(M;t) = s_0 \cos(\omega t - k r_1) & k = \frac{2\pi}{\lambda}. \\ y_2(M;t) = s_0 \cos(\omega t - k r_2) \end{cases}$$

$\varphi_{2/1} = k(r_1 - r_2) = -k(r_2 - r_1)$

$r_1 = O_1M = \sqrt{(x - \frac{a}{2})^2 + D^2}$ $\Rightarrow$ $r_1^2 = x^2 - ax + \frac{a^2}{4} + D^2$

$r_2 = O_2M = \sqrt{(x + \frac{a}{2})^2 + D^2}$ $\qquad$ $r_2^2 = x^2 + ax + \frac{a^2}{4} + D^2$ $\rightarrow$ $r_2^2 - r_1^2 = 2ax$.

et $r_2^2 - r_1^2 = (r_2 - r_1)\underbrace{(r_2 + r_1)}_{\simeq 2D}$ $\Rightarrow$ $r_2 - r_1 = \frac{2ax}{2D} = \frac{ax}{D}$.

donc $\varphi_{2/1} = -k \cdot \frac{ax}{D} = \boxed{-\frac{2\pi}{\lambda} \cdot \frac{ax}{D} = \varphi_{2/1}} = \varphi_2 - \varphi_1)$

3-

$y_1(M,t) + y_2(M,t) = s_0 \cos(\omega t + \varphi_1) + s_0 \cos(\omega t + \varphi_2) = \underbrace{2 s_0 \cos\left(\frac{\varphi_2 - \varphi_1}{2}\right)}_{\text{Amplitude}} \cos\left(\omega t + \frac{\varphi_1 + \varphi_2}{2}\right)$

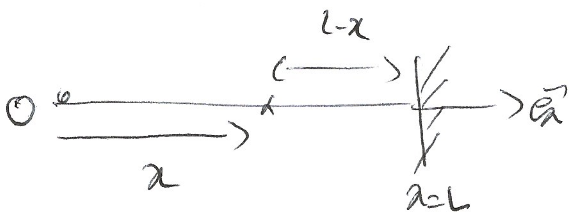
4- Interférences constructives :

$2 s_0 \cos\left(\frac{\varphi_2 - \varphi_1}{2}\right) = \pm 2 s_0$ $\rightarrow$ $\cos\left(\frac{\varphi_2 - \varphi_1}{2}\right) = \pm 1$ $\rightarrow$ $\frac{\varphi_2 - \varphi_1}{2} = p\pi$ $\rightarrow$ $\boxed{\varphi_2 - \varphi_1 = 2p\pi}$

or $\varphi_2 - \varphi_1 = -\frac{2\pi}{\lambda} \frac{ax}{D} = 2p\pi$ $\rightarrow$ $\frac{ax}{D} = -p\lambda$ $\rightarrow$ $\boxed{x = -p \frac{\lambda D}{a}}$

$\boxed{\Delta x = i = \frac{\lambda D}{a}}$

ex2 réflexion d'une onde acoustique.



1) $p_i(0,t) = p_0 \cos 2\pi f t$

$p_i(x;t) = p_0 \cos 2\pi f\left(t - \frac{x}{c}\right) = p_0 \cos 2\pi f t - kx$.

$p_i(L,t) = p_0 \cos(2\pi f t - kL)$ $\quad p_r(L;t) = p_0 \cos(2\pi f t - kL)$

$p_r(x,t) = p_0 \cos 2\pi f\left(t - \left(\frac{L-x}{c}\right) - kL\right) = p_0 \cos(2\pi f t + kx - 2kL)$.

$\Delta\varphi = \varphi_r - \varphi_i = kx - 2kL - (-kx) = 2k(x-L) = \frac{4\pi}{\lambda}(x-L)$.

2) $p_i(x,t) + p_r(x,t) = p_0 \left[\cos(2\pi f t + \varphi_i) + \cos(2\pi f t + \varphi_r)\right]$

$$\underbrace{2p_0 \cos \frac{\varphi_r - \varphi_i}{2}}_{\text{amplitude}} \cos\left(2\pi f t + \frac{\varphi_r + \varphi_i}{2}\right)$$

Interférences destructives si $\cos \frac{\varphi_r - \varphi_i}{2} = 0$

$\frac{\varphi_r - \varphi_i}{2} = (2p+1)\frac{\pi}{2} \Rightarrow \varphi_r - \varphi_i = (2p+1)\pi \Rightarrow \frac{4\pi}{\lambda}(L-x) = (2p+1)\pi$

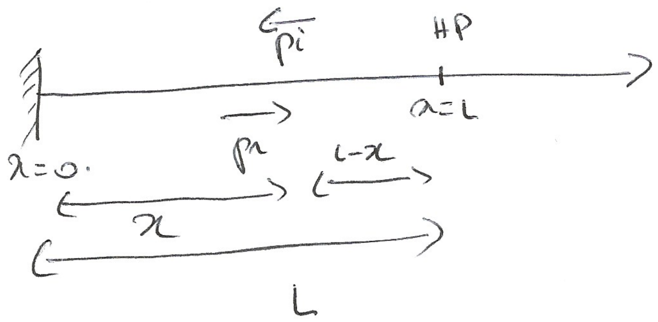
$L - x = \frac{(2p+1)\lambda}{4} = \left(p + \frac{1}{2}\right) \cdot \frac{\lambda}{2} \Rightarrow x = L - \left(p + \frac{1}{2}\right)\frac{\lambda}{2}$

$p = 0 \Rightarrow x_1 = L - \frac{\lambda}{2}$

$p = 1 \rightarrow x_2 = L - \frac{3\lambda}{2}$

⋮

ex3 Tube de Kundt.



1°] $p_i(L;t) = A_0 \cos \omega t$.

2°] $p_i(x,t) = p_i\left(L; t - \left(\frac{L-x}{c}\right)\right)$

$= A_0 \cos \omega\left(t - \left(\frac{L-x}{c}\right)\right)$

$= A_0 \cos(\omega t + kx - kL)$

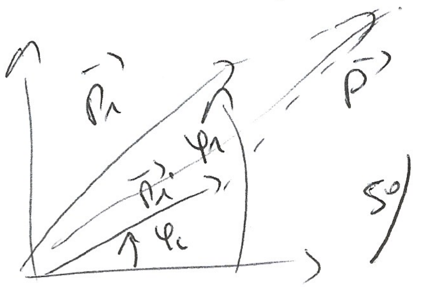
3°) $p_i(x=0,t) = A_0\cos(\omega t - kL) \Rightarrow p_r(0,t) = A_0 r\cos(\omega t - kL)$

$p_r(x,t) = p_r\left(0, t-\frac{x}{c}\right) = A_0 r \cos\left(\omega\left(t-\frac{x}{c}\right) - kL\right) = A_0 r\cos(\omega t - kx - kL)$

$\varphi_i(x) = kx - kL$ $\qquad \varphi_r(x) = -kx - kL$.

$p(x,t) = p_i(x,t) + p_r(x,t) \Rightarrow$ Methode de Fresnel.

4°) $p_i(x,t) \longrightarrow \vec{P_i}\ \Big|\ \|\vec{P_i}\| = A_0,\ (\vec{Ox}, \vec{P_i}) = \varphi_i$ $\qquad p_r(x,t) \longrightarrow \vec{P_r}\ \Big|\ \|\vec{P_r}\| = r A_0,\ (Ox, \vec{P_r}) = \varphi_r$.



$\vec{P}^2 = \left(\vec{P_i} + \vec{P_r}\right)^2 = P_i^2 + P_r^2 + 2P_iP_r\cos(\varphi_r - \varphi_i)$

5°) $A^2 = A_0^2 + r^2A_0^2 + 2rA_0^2\cos(\varphi_i - \varphi_r)$.

$A_{max} = A_0^2 + r^2A_0^2 + 2rA_0^2 = A_0^2(1+r)^2$ $\qquad \alpha = \frac{A_{max}}{A_{min}} = \left(\frac{1-r}{1+r}\right)^2 \Rightarrow$

$A_{min} = A_0^2 + r^2A_0^2 - 2rA_0^2 = A_0^2(1-r)^2$

$1 - r = (1+r)\alpha \Rightarrow 1 - \alpha = (\alpha+1)r \Rightarrow \boxed{r = \frac{1-\alpha}{1+\alpha}}$.

Ex5 :

Etude des modes propres d'une corde.

1) $f_2 = 2 f_0$ $\qquad$ d'où $\frac{f_2}{2}$ doit être égale à $\frac{f_3}{3}$

$f_3 = 3 f_0$

$\frac{19}{2} = 9,5$ et $\frac{28}{3} = 9,33$ Mesures compatibles

$\boxed{f_0 \simeq 9\ \text{Hz} = f_3 - f_2}$ $(9,4\ \text{Hz})$ $\qquad f_4 = 4\cdot f_0$ $(38\ \text{Hz})$.

$L = 117\ \text{cm}$ $\qquad f_0 = \frac{c}{2L} \Rightarrow c = 2Lf_0 = 1,17\cdot 2\cdot 9,4 = 22\ m\,s^{-1}$

$\boxed{T = mg = 0,25\ N}$

$c = \sqrt{\frac{T}{\mu}} \Rightarrow \mu = \frac{T}{c^2} = \frac{0,25}{22^2} = 5,2\times 10^{-4}\ kg\,m^{-1}$

$= \boxed{0,52\ g\,m^{-1} = \mu}$

*4 Ondes stationnaires.

1] Onde stationnaire.

a) $y_i(x,t) = a\cos(\omega t - kx)$.

$y_r(x,t) = a\cos(\omega t + kx + \varphi)$.

$$y(x,t) = y_i(x,t) + y_r(x,t) = \boxed{2a\cos\left(kx + \frac{\varphi}{2}\right)\cos\left(\omega t + \frac{\varphi}{2}\right) = y(x}$$

L'amplitude $\boxed{A(x) = 2a\cos\left(kx + \frac{\varphi}{2}\right)}$ dépend de la variable x.

b) Les nœuds sont des points de vibration nulle.

$\Rightarrow A(x_N) = 0 \Rightarrow k(x_N)_p + \frac{\varphi}{2} = (2p+1)\frac{\pi}{2} \Rightarrow \boxed{(x_N)_p = \left((2p+1)\frac{\pi}{2} - \frac{\varphi}{2}\right)\cdot\frac{1}{k}}$

$\boxed{(x_N)_{p+1} - (x_N)_p = \frac{\pi}{k} = \frac{\pi\cdot\lambda}{2\pi} = \frac{\lambda}{2}}$. Distance entre 2 nœuds

Distance entre 2 ventres.

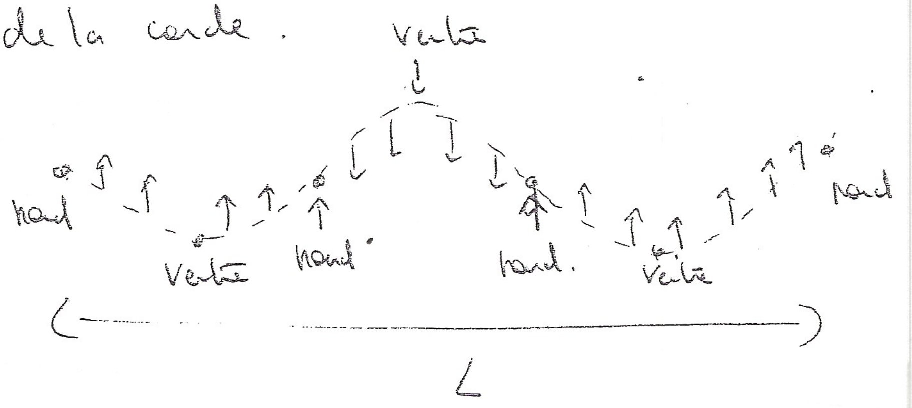
Les ventres sont les points de vibration maximale.

$\Rightarrow \cos\left(kx_V + \frac{\varphi}{2}\right) = \pm 1 \Rightarrow$

$k(x_V)_p + \frac{\varphi}{2} = p\cdot\pi \Rightarrow \boxed{(x_V)_p = \left(p\pi - \frac{\varphi}{2}\right)\cdot\frac{1}{k}}$

La distance entre 2 ventres est $\boxed{(x_V)_{p+1} - (x_V)_p = \frac{\pi}{k} = \frac{\lambda}{2}}$.

Allure de la corde.



La vibration étant rapide, l'œil perçoit des fuseaux.

c) L'onde stationnaire est stable lorsque les fuseaux sont bien marqués. Il existe alors une fréquence de vibration qui vérifie $L = n \cdot \frac{\lambda}{2} = n \frac{c}{2 f_n} \Rightarrow \boxed{f_n = n \frac{c}{2L}}$

La fréquence fondamentale est obtenue pour $n = 1 \Rightarrow \boxed{f_1 = \frac{c}{2L}}$

Question 2

$c = \sqrt{\frac{T}{\mu}}$

$f_n = n \cdot f_1$

$f_{n+1} = (n+1) f_1$

$\Rightarrow f_1 = f_{n+1} - f_n = 112 \text{ Hz}$

$n = 3$

$f_1 = \frac{c}{2L} \Rightarrow c = f_1 \cdot 2L = 112 \cdot 2 \cdot 0,6 = \boxed{134,4 \text{ m s}^{-1} = c.}$

$c = \sqrt{\frac{T}{\mu}} \rightarrow \boxed{T = c^2 \cdot \mu}$ AN $T = (134,4)^2 \cdot 1,8 \cdot 10^{-3} = \boxed{32,5 \text{ N}}$

b)

$c = \sqrt{\frac{T}{\mu}}$ $c' = \sqrt{\frac{T'}{\mu'}}$ avec $T' = \frac{T}{2}$ et $\mu' = 4\mu$.

Rq $\mu = \frac{m}{L} = \frac{\rho \cdot V}{L}$ où ρ est la masse volumique et

V le volume de la corde de rayon R et de longueur L.

d'où $V = \pi R^2 \cdot L \Rightarrow \mu = \rho \cdot \pi R^2$ d'où $\mu' = \rho \pi (2R)^2$
$= 4 \rho \pi R^2$.

d'où $c' = \sqrt{\frac{T}{2 \cdot 4 \mu}} = \frac{1}{2\sqrt{2}} \cdot \sqrt{\frac{T}{\mu}} \Rightarrow \boxed{c' = \frac{1}{2\sqrt{2}} \cdot c}$

exo6

· surpression $p(x,t) = \underbrace{P(x,t)}_{\text{pression}} - P_0$

$\uparrow$ pression atmosphérique.

Instrument ouvert $p(x,t) = 0$ à son extrémité → nœud de pression.

Instrument fermé : amplitude de $p(x,t)$ max → ventre de pression.

a] flûte ouverte à ses 2 extrémités.

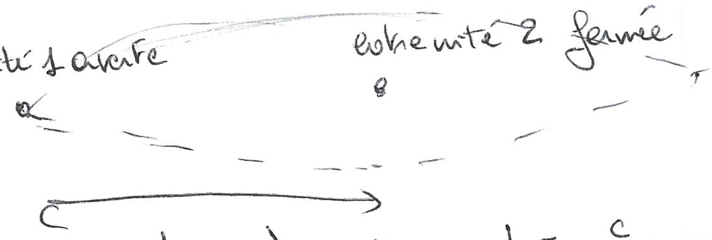
longueur L de la flute

$L = n \dfrac{\lambda_n}{2}$ fondamental correspond à $n=1$

$L = \dfrac{\lambda}{2} = \dfrac{c}{2f}$ → $\boxed{f_1 = \dfrac{c}{2L}}$

AN $f_1 = 330$ Hz et $c = 340\ \mathrm{m\,s^{-1}}$ → $\boxed{L = 51{,}5\ \text{cm}.}$

b] Clarinette.



ventre

extrémité 2 fermée

extrémité 1 ouverte

nœud

$L = \dfrac{\lambda}{4}$ → $L = \dfrac{c}{4 f_1}$ → $\boxed{f_1 = \dfrac{c}{4L}.}$

Pour n quelconque $L = n\dfrac{\lambda}{2} + \dfrac{\lambda}{4} = \overbrace{(2n+1)}^{p} \cdot \dfrac{\lambda_p}{4}$

donc p impair ⇒ $L = p \cdot \dfrac{c}{f_p \cdot 4}$ → $\boxed{f_p = p \cdot \dfrac{c}{4L}.}$ $p = 1, 3, 5, \ldots$

c) $(f_1)_{\text{clarinette}} = \dfrac{(f_1)_{\text{flute}}}{2} = 165$ Hz la clarinette produit un son + grave.

d) $\Delta f_{\text{flute}} = f_{n+1} - f_n = (n+1) - n \cdot \dfrac{c}{2L} = \dfrac{c}{2L}$ $(\Delta f)_{\text{clarinette}} = [(p+2) - p]\dfrac{c}{4L} = \dfrac{2c}{4L} = \dfrac{c}{2L}$