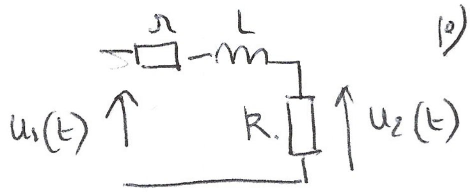
Correction TD Filtrage.



1°) BF $\rightsquigarrow$ $\Rightarrow$ — $\quad u_2 = \dfrac{R}{R+r} u_1$
HF $\rightsquigarrow$ $\Rightarrow$ $\quad u_2 = R\underset{0}{i} = 0$
} Filtre passe bas.

2°] $u_1(t) = U_1 \cos\omega t \Rightarrow \underline{u_1} = U_1$
$u_2(t) = U_2 \cos(\omega t + \varphi) \Rightarrow \underline{u_2} = U_2 e^{j\varphi}$.

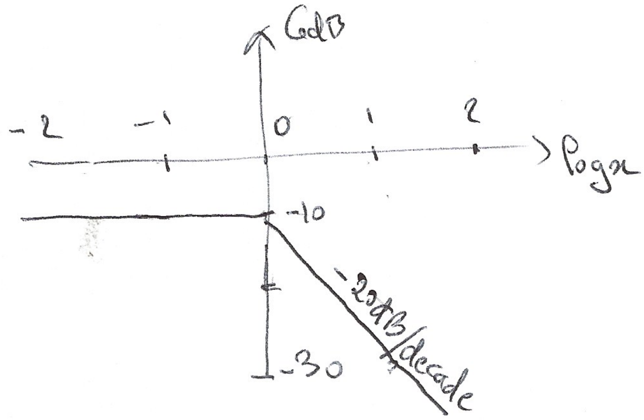
$$\underline{H} = \frac{\underline{u_2}}{\underline{u_1}} = \frac{R}{R+r+jL\omega} = \frac{\frac{R}{R+r}}{1+j\frac{L\omega}{R+r}}$$

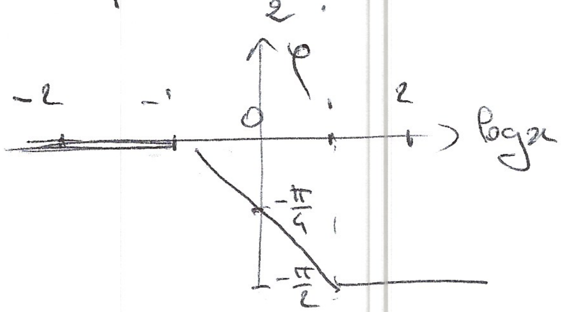
Forme canonique : $\underline{H} = \dfrac{H_0}{1+j\frac{\omega}{\omega_c}} = |\underline{H}|e^{j\varphi}$ $\quad H_0 = \dfrac{R}{R+r} = H(\omega=0)$

$\omega_c = \dfrac{R+r}{L}$ = pulsation de coupure.

BF $\omega < 0{,}1\omega_c$ $\quad \underline{H} \to H_0$ $\quad$ GdB $\to Y_{BF} = 20\log H_0$
$\varphi \to 0$

HF $\omega > 10\,\omega_c$ $\quad \underline{H} \to \dfrac{H_0}{j\frac{\omega}{\omega_c}} \Rightarrow$ GdB $\to Y_{HF} = 20\log H_0 - 20\log x$.
$\varphi \to -\dfrac{\pi}{2}$.





3°] $r = 10\Omega \quad R = 90\Omega \Rightarrow \dfrac{R}{R+r} = \dfrac{90}{100} = 0{,}9 \Rightarrow 20\log H_0 = 20\log 0{,}9$
$=$

$$f_c = \frac{\omega_c}{2\pi} = \frac{R+r}{L\cdot 2\pi} = \frac{100}{10^{-2}\cdot 2\pi}$$

$$f_c = \frac{10^4}{2\pi} = 1592\ \text{Hz}.$$

× $f_1 = 100$ Hz $\Rightarrow \dfrac{f}{f_c} = 0{,}06 \Rightarrow U_s(t) = U_1 \cdot 0{,}9\cos(2\pi f_1 t + 0)$

× $f_2 = 10$ kHz $\Rightarrow \dfrac{f}{f_c} = \cancel{\ }\ 6{,}3 \Rightarrow U_s(t) = \dfrac{U_1\cdot 0{,}9}{\sqrt{1+(6{,}3)^2}}\cos(2\pi f_2 t - \arctan 6{,}3)$.

$\Rightarrow U_s(t) = 0{,}14\,U_e \cos(2\pi f_2 t - 1{,}4)$

4°] $T = 0{,}1$ ms $= 10^{-4}$ s $= f_3 = 1$ kHz.

$u_e(t)$ signal carré de fréquence fondamentale $f_0 = 1$ kHz $\Rightarrow x_1 = \dfrac{1}{1{,}6}$

$3f_0 = 3$ kHz $\quad x_3 = \dfrac{3}{1{,}6}$.

$5f_0 = 5$ kHz $\quad x_5 = \dfrac{5}{1{,}6}$

$$u_s(t) = \frac{4U}{\pi}\left[\frac{0{,}9}{\sqrt{1+\left(\frac{1}{1{,}6}\right)^2}}\cos\left(2\pi f_0 t - \operatorname{arctg}\left(\frac{1}{1{,}6}\right)\right)\right]$$

$$+ \frac{4U}{3\pi}\left[\frac{0{,}9}{\sqrt{1+\left(\frac{3}{1{,}6}\right)^2}}\cos\left(2\pi(3f_0 t) - \operatorname{arctan}\left(\frac{3}{1{,}6}\right)\right)\right]$$

$$+ \frac{4U}{5\pi}\left[\frac{0{,}9}{\sqrt{1+\left(\frac{5}{1{,}6}\right)^2}}\cos\left(2\pi(5f_0 t) - \operatorname{arctan}\frac{5}{1{,}6}\right)\right]$$

$$+ \frac{4U}{7\pi}\left[\frac{0{,}9}{\sqrt{1+\left(\frac{7}{1{,}6}\right)^2}}\cos\left(2\pi(7f_0 t) - \operatorname{arctg}\frac{7}{1{,}6}\right)\right]$$

Ex2 recepteur radio.

1) Filtre passe bande → u_s en sortie de R.

2) $\underline{H} = \frac{\underline{u_s}}{\underline{u_e}} = \frac{R}{R + j\left(L\omega - \frac{1}{C\omega}\right)} = \frac{1}{1 + j\left(\frac{L\omega}{R} - \frac{1}{RC\omega}\right)}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$\frac{L}{R} = \frac{Q}{\omega_0} \Rightarrow$ $\boxed{Q = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}}$

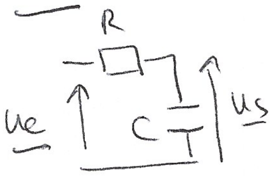
$$\begin{cases} f_0 = \frac{1}{2\pi\sqrt{LC}} \\ \text{et } Q = \frac{f_0}{\Delta f} \Rightarrow \Delta f = \frac{f_0}{Q} = \boxed{\frac{R}{2\pi \cdot L} = \Delta f} \end{cases}$$ ne depend pas de C

$f_1 < f_0 < f_2$ $\quad f_1 = 150\,Hz$ $\quad f_2 = 300\,Hz$ $\Rightarrow C_2 < C < C_1$.

$C_2 = \frac{1}{4\pi^2 f_2^2 \cdot L} = 2{,}8\cdot 10^{-10}\,F$

$C_1 = \frac{1}{4\pi^2 f_1^2 \cdot L} = 1{,}1\cdot 10^{-9}\,F$.

Ex3



1] $\underline{H_1} = \frac{\underline{u_s}}{\underline{u_e}} = \frac{\frac{1}{jC\omega}}{\frac{1}{jC\omega} + R} = \frac{1}{1 + jRC\omega}$ passe bas 1er ordre

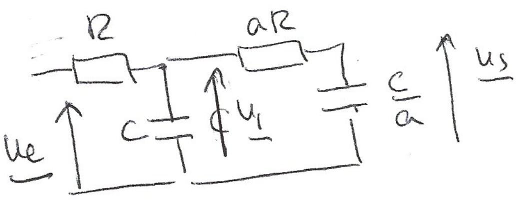
$= \frac{H_0}{1 + j\frac{\omega}{\omega_0}}$.

$C \to \frac{C}{a}$

$R \to aR$

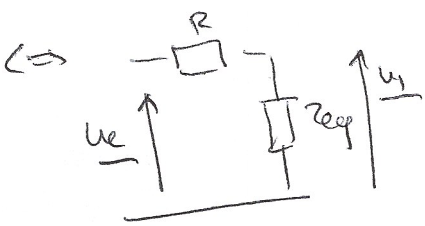
$\frac{\underline{u_s}}{\underline{u_e}} = \frac{\frac{a}{jC\omega}}{\frac{a}{jC\omega} + aR} = \frac{1}{1 + jRC\omega}$ $\underline{H}$ n'est pas modifié.

2]



$\frac{\underline{u_s}}{\underline{u_e}} = \frac{\underline{u_s}}{\underline{u_1}} \cdot \frac{\underline{u_1}}{\underline{u_e}} = \frac{1}{1 + jRC\omega} \cdot \frac{\underline{u_1}}{\underline{u_e}}$.

⇔



$\frac{\underline{u_1}}{\underline{u_e}} = \frac{Z_{eq}}{R + Z_{eq}} = \frac{1}{1 + RY_{eq}} = \frac{1}{1 + R\left[jC\omega + \frac{1}{aR + \frac{a}{jC\omega}}\right]}$

$$\frac{\underline{u_1}}{\underline{u_e}} = \frac{1}{1+jRC\omega+\frac{jRC\omega}{a(1+jRC\omega)}} = \frac{a(1+jRC\omega)}{a[1+jRC\omega]^2+jRC\omega}$$

$$= \frac{1+jRC\omega}{1+\left(2+\frac{1}{a}\right)jRC\omega+(jRC\omega)^2}$$

$$\Rightarrow \boxed{\underline{H} = \frac{1}{1+\left(2+\frac{1}{a}\right)jRC\omega+(jRC\omega)^2} = \frac{H_0}{1+j\frac{\omega}{Q\omega_0}+\left(j\frac{\omega}{\omega_0}\right)^2}}$$

Filtre passe bas 2nd ordre.

$\omega_0 = \frac{1}{RC}$ $\quad RC\left(2+\frac{1}{a}\right) = \frac{1}{Q\omega_0} \Rightarrow Q = \frac{1}{2+\frac{1}{a}} < \frac{1}{2}$ donc $Q < \frac{1}{\sqrt{2}}$

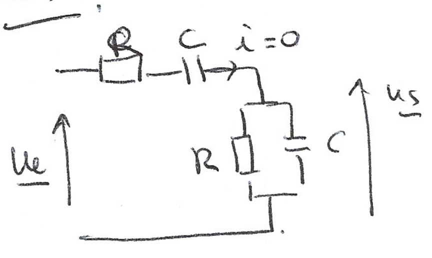
pas de resonance

eq. diff associée:

$$R^2C^2\frac{d^2u_s}{dt^2} + \left(2+\frac{1}{a}\right)RC\frac{du_s}{dt} + u_s = u_e(t).$$

Si $a >> 1$ $\quad Q = \frac{1}{2}$ et $\underline{H} = (\underline{H_1})^2$ car $Z_e \uparrow \infty$.

Ex4.



BF $-||- \Leftrightarrow -/\;-$ $\quad u_s = Ri$ et $i=0 \to u_s = 0$

HF $-||- \Leftrightarrow -$ $\quad u_s = 0$

Passe bande.

$$\frac{\underline{u_s}}{\underline{u_e}} = \frac{Z_2}{Z_1+Z_2} = \frac{1}{1+Z_1Y_2} = \frac{1}{1+\left(R+\frac{1}{jC\omega}\right)\left(\frac{1}{R}+jC\omega\right)}$$

$$= \frac{1}{1+1+jRC\omega+\frac{1}{jRC\omega}+1} = \frac{1}{3+j\left(RC\omega-\frac{1}{RC\omega}\right)} = \frac{1/3}{1+j\left(\frac{RC\omega}{3}-\frac{1}{3RC\omega}\right)}$$

$$= \frac{H_0}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)} \Rightarrow H_0 = \frac{1}{3}$$

$$\begin{cases} \frac{RC}{3} = \frac{Q}{\omega_0} \\ \frac{1}{3RC} = Q\cdot\omega_0 \end{cases} \Rightarrow \omega_0 = \frac{1}{RC}, \quad Q = \frac{1}{3}$$

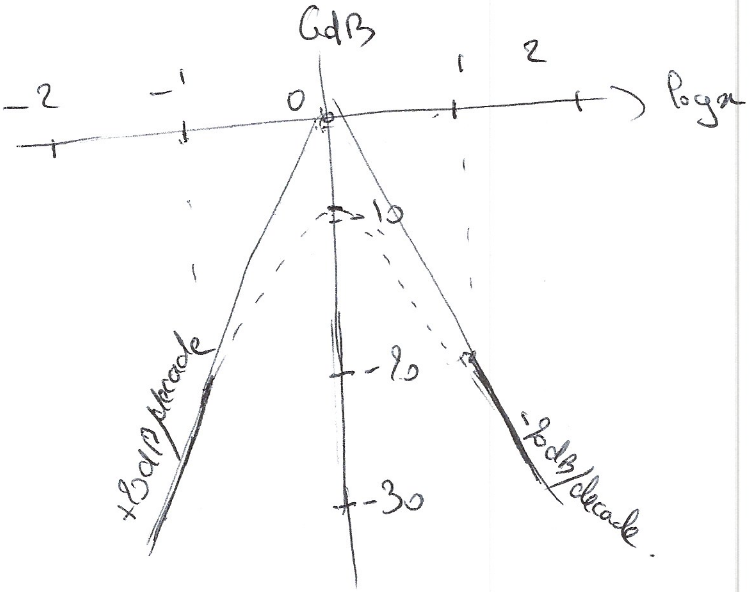
$G_{max_{dB}} = 20\log H_0 = 20\log\frac{1}{3} = -9{,}5\,dB$

$\underline{H} = \dfrac{H_0}{1+jQ(x-\frac{1}{x})} \xrightarrow{BF} \dfrac{H_0}{-j\frac{Q}{x}} \Rightarrow j\dfrac{H_0}{Q}\cdot x \Rightarrow \begin{cases} G_{dB} \to y_{BF} = 20\log\frac{H_0}{Q} + 20\log x. \\ \varphi \to \frac{\pi}{2} \end{cases}$

$\underline{H} = \dfrac{H_0}{1+jQ(x-\frac{1}{x})} \xrightarrow{BF} \dfrac{H_0}{jQx} = -j\dfrac{H_0}{Q}\cdot\dfrac{1}{x}.$

$\Rightarrow \begin{cases} G_{dB} \to y_{HF} = 20\log\frac{H_0}{Q} - 20\log x. \\ \varphi \to \frac{-\pi}{2}. \end{cases}$

$I \left| \begin{array}{l} \log x = 0 \quad (x=1) \\ 20\log\frac{H_0}{Q} = 1. \end{array} \right.$



5°] $\omega_0 = \dfrac{1}{RC} = 2\cdot10^3\,rad\,s^{-1}.$

$\omega_1 = 2\cdot10^2\,rad\,s^{-1} \Rightarrow x_1 = \dfrac{\omega_1}{\omega_0} = 0{,}1$

$\omega_2 = 10\,\omega_1 = 2\cdot10^3\,rad\,s^{-1} \quad x_2 = 1.$

$\omega_3 = 100\,\omega_1 = 2\cdot10^4\,rad\,s^{-1} \quad x_3 = 10.$

Rappel $e(t) = \Sigma\, E_n \cos(\omega_n t + \varphi_n).$

$s(t) = \Sigma\, E_n\cdot|\underline{H}(\omega_n)| \cos\left(\omega_n t + \varphi_n + \arg\underline{H}(\omega_n)\right).$

$\arg\underline{H}(\omega_n) = -\arctan Q\left(\dfrac{\omega_n}{\omega_0} - \dfrac{\omega_0}{\omega_n}\right).$

$$s(t) = \frac{E_0}{10}\cos(\omega_1 t + 1{,}2) + \frac{E_0}{3}\cos(10\omega_1 t) + \frac{E_0}{10}\cos(100\omega_1 t + 1{,}2)$$

__eos__ lecture ~~de~~ diagrammes de Bode.

$$e(t) = \sum_n E_n \cos(\omega_N t + \varphi_n)$$

$$s(t) = \sum |\underline{H}(\omega_n)| \cdot E_n \cos(\omega_n t + \varphi_n + \arg \underline{H}(\omega_N)).$$

où $|\underline{H}(\omega_N)| = 10^{G_{dB}(\omega_n)/20}$ ~~et~~

__1° filtre passe haut__ $f_0 = 10\,kHz$. sans resonance.

$f_1 = 1\,kHz \Rightarrow x_1 = \frac{f_1}{f_0} = 0{,}1$

$e_1(t)$ frequence $f_1 \to x_1 = 0{,}1$

$e_2(t)$ $10 f_1 = f_2 \to x_2 = 1$

$e_3(t)$ $100 f_1 = f_3 = x_3 = 10$

$e(t) = e_0 + e_1(t) + e_2(t) + e_3(t)$

$s(t) = s_0 + s_1(t) + s_2(t) + s_3(t)$

$s_0 = 0$

$s_1 \simeq 0$ car attenuation de 40 dB $\Rightarrow$ division de l'amplitude par 100.

$$s_2(t) = E_0 \cdot \underbrace{\underbrace{10^{-G_{dB}/20}}_{10^{-\frac{1}{2}}}}_{\simeq 0{,}3} \cos\left(f_2 t + \frac{\pi}{4} + \frac{\pi}{2}\right) = 0{,}3\,E_0 \cos\left(f_2 t + \frac{3\pi}{4}\right).$$

$s_3(t) \simeq e_3(t) =$

__2° filtre passe haut resonant__. $f_r = 100\,Hz$.

Composante continue coupée $\to s(t) = e(t) - e_0$

3° filtre. rejection $f_0 = 1\,kHz$.

$s(t) = e(t) - e_1(t)$.

5° Filtre

passe bas 1° ordre. $f_0 = 100\ Hz$. $H_0 = 1$.

$s_0 = e_0$.

$e_1(t)$ de frequence $10 f_0$. Amplitude de $s_1(t)$ divisée par 10.
$\varphi_H = -\frac{\pi}{2}$.

$e_2(t)$ de frequence $100 f_0$ Amplitude de $s_2(t)$ divisée par 100, $\varphi_H = -\frac{\pi}{2}$
$e_3(t)$ " " $1000 f_0$ " " 1000 "

$$s(t) = E_0 + \frac{E_0}{10}\cos\left(\omega t - \frac{\pi}{2}\right) + \frac{E_0}{100}\cos\left(10\omega t - \frac{\pi}{4}\right) + \frac{E_0}{1000}\cos\left(100\omega t - \frac{5\pi}{6}\right).$$