Correction TD circuits électriques en RSF.

Ex1 Calcul d'impédances.

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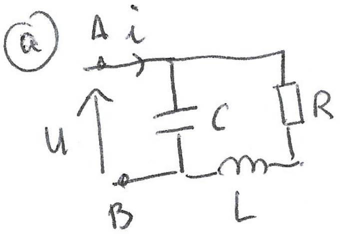


(R série L) // C $\Rightarrow \underline{Y} = jC\omega + \dfrac{1}{R+jL\omega} = \dfrac{1-LC\omega^2+jRC\omega}{R+jL\omega}$.

$$\underline{Z}_{AB} = \frac{R+jL\omega}{1-LC\omega^2+jRC\omega} \Rightarrow \boxed{|\underline{Z}| = \left[\frac{R^2+L^2\omega^2}{(1-LC\omega^2)^2+R^2C^2\omega^2}\right]^{\frac{1}{2}}}$$

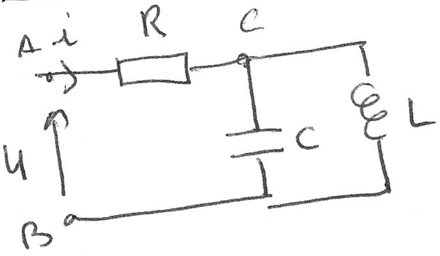
$1-LC\omega^2 > 0$ pour $\omega < \dfrac{1}{\sqrt{LC}}$

< 0 pour $\omega > \dfrac{1}{\sqrt{LC}}$ $\Rightarrow$ Il est préférable d'écrire $\underline{Z}$ sous la forme :

$$\underline{Z} = \frac{R+jL\omega}{jC\omega\left[R+\frac{1-LC\omega^2}{jC\omega}\right]} = \frac{R+jL\omega}{jC\omega\left[R+j\left(L\omega-\frac{1}{C\omega}\right)\right]} = \frac{|N|e^{j\varphi_N}}{|D|e^{j\varphi_D}} = |Z|e^{j\varphi}.$$

$$\Rightarrow \boxed{\varphi = \varphi_N - \varphi_D = \operatorname{arctg}\frac{L\omega}{R} - \frac{\pi}{2} - \operatorname{arctg}\left(\frac{L\omega-\frac{1}{C\omega}}{R}\right)}$$

ⓑ



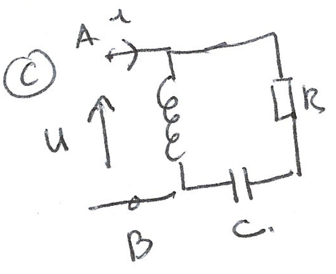
C//L en série avec R.

$Z_{AB} = R + \dfrac{1}{Y_{CB}}$ et $Y_{CB} = jC\omega + \dfrac{1}{jL\omega}$

$$= R + \frac{1}{jC\omega + \frac{1}{jL\omega}} = R + \frac{jL\omega}{1-LC\omega^2}.$$

$$\boxed{|Z_{AB}| = \left[R^2 + \frac{L^2\omega^2}{(1-LC\omega^2)^2}\right]^{\frac{1}{2}}} \text{ et } \boxed{\varphi = \operatorname{arctg}\frac{L\omega}{R(1-LC\omega^2)}}$$

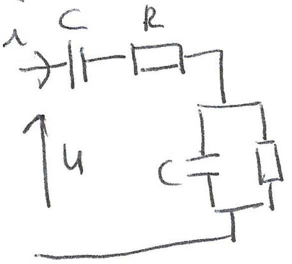
ⓒ



(R série avec C) // L. $Z = \dfrac{\left(R+\frac{1}{jC\omega}\right)jL\omega}{\left(R+\frac{1}{jC\omega}\right)+jL\omega} = \dfrac{(1+jRC\omega)\cdot\cancel{j}L\cancel{\omega}}{\left[R+j\left(L\omega-\frac{1}{C\omega}\right)\right]\cancel{j}C\cancel{\omega}}$

$$\boxed{|Z| = \frac{L}{C}\frac{\sqrt{1+R^2C^2\omega^2}}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}}$$

$Z = |Z|e^{j\varphi}$ et $\boxed{\varphi = \operatorname{arctg} RC\omega - \operatorname{arctg}\left(\dfrac{L\omega-\frac{1}{C\omega}}{R}\right)}$



$$Z = Z_1 + Z_2$$

avec $Z_1 = R + \frac{1}{jC\omega} = \frac{1 + jRC\omega}{jC\omega}$

$$Z_2 = \frac{R \cdot \frac{1}{jC\omega}}{R + \frac{1}{jC\omega}} = \frac{R}{1 + jRC\omega}$$

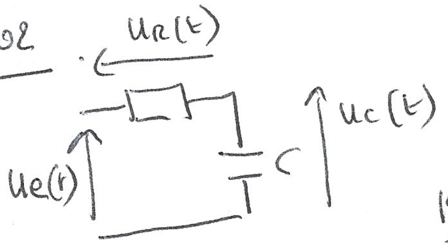
d'où $Z = \frac{1 + jRC\omega}{jC\omega} + \frac{R}{1 + jRC\omega} = \frac{1 + 2jRC\omega + (jRC\omega)^2 + jRC\omega}{jC\omega(1 + jRC\omega)}$.

$$= \frac{1 + 3jRC\omega + (jRC\omega)^2}{jC\omega(1 + jRC\omega)} = \frac{R\left[3 + j\left(RC\omega - \frac{1}{RC\omega}\right)\right]}{1 + jRC\omega}.$$

$$\boxed{|Z| = \frac{R\sqrt{3 + \left(RC\omega - \frac{1}{RC\omega}\right)^2}}{\sqrt{1 + R^2C^2\omega^2}}.}$$

$$\varphi = \arctan\left(\frac{1}{3}\left(RC\omega - \frac{1}{RC\omega}\right)\right) - \arctan RC\omega$$

ex2



$u_e(t) = E\cos\omega t \rightarrow \underline{u_e}(t) = Ee^{j\omega t}$

$\underline{u_e} = E$

1°) . $u_e(t) = Ri(t) + u_c$

$= RC\frac{du_c}{dt} + u_c \Rightarrow \frac{du_c}{dt} + \frac{u_c}{RC} = \frac{u_e(t)}{RC}$.

a] en RSF $u_s(t) = U_s(\omega)\cos(\omega t + \varphi(\omega))$. et $\underline{u_s} = U_s(\omega)e^{j\varphi(\omega)}$.

$\rightarrow \underline{u_s}(t) = U_s(\omega)\exp j(\omega t + \varphi(\omega))$

b] $\frac{\underline{u_c}}{\underline{u_e}} = \frac{\frac{1}{jC\omega}}{R + \frac{1}{jC\omega}} = \frac{1}{1 + jRC\omega}$ $\Rightarrow$ $\boxed{\underline{u_s} = \frac{\underline{u_e}}{1 + jRC\omega}.}$ par le diviseur de tension.

ou par la loi des mailles : $\underline{u_e} = R\underline{i} + \underline{u_c}$ et $\underline{i} = \frac{\underline{u_e}}{R + \frac{1}{jC\omega}}$.

$\underline{u_e}\left(1 - \frac{jRC\omega}{1 + jRC\omega}\right) = \underline{u_c} \rightarrow \boxed{\underline{u_c} = \frac{\underline{u_e}}{1 + jRC\omega}}$.

$U_c(\omega) = \frac{E}{\sqrt{1 + R^2C^2\omega^2}}$ et $\varphi = -\arctan(RC\omega)$

$\Rightarrow u_c(t) = \frac{E}{\sqrt{1 + R^2C^2\omega^2}}\cos(\omega t - \arctan RC\omega)$

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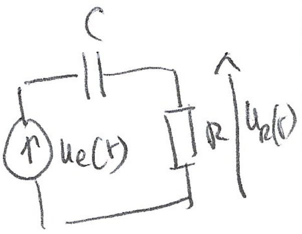
pour $\omega = \omega_0$ $\boxed{u_C(t) = \dfrac{E}{\sqrt{2}} \cos\left(\omega_0 t - \dfrac{\pi}{4}\right)}$

$\omega = 2\omega_0$ $u_C(t) = \dfrac{E}{\sqrt{5}} \cos\left(2\omega_0 t - \operatorname{arctg} 2\right)$



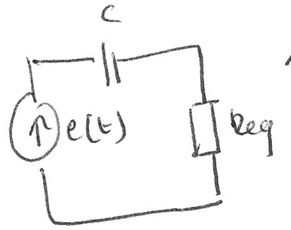
3°] $u_R(t) = U_R(\omega) \cos(\omega t + \varphi(\omega)) \Rightarrow \underline{u_R} = U_R(\omega) e^{j\varphi(\omega)}$

$\dfrac{\underline{u_R}}{\underline{u_e}} = \dfrac{R}{R + \dfrac{1}{jC\omega}} \Rightarrow \dfrac{jRC\omega}{1 + jRC\omega} \Rightarrow U_R(\omega) = \dfrac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}}$

$\varphi = \dfrac{\pi}{2} - \operatorname{arctg} RC\omega.$

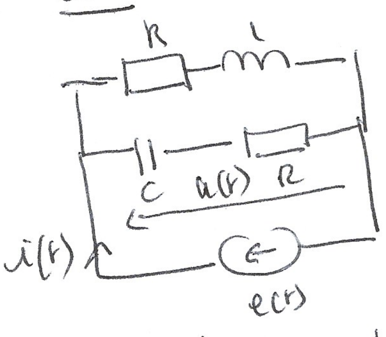
$$\boxed{u_R(t) = \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}} \cos\left(\omega t + \frac{\pi}{2} - \operatorname{arctg} RC\omega\right)}$$

4°] $R_{eq} = \dfrac{RR_u}{R + R_u}$



même structure que ds la question 3 ; il suffit de remplacer R par $R_{eq} = \dfrac{RR_u}{R + R_u}$.

ex3 Calcul de déphasage



$u(t) = e(t) = E \cos(\omega t + \varphi_1) \rightarrow \underline{u} = E e^{j\varphi_1}$

$i(t) = I \cos(\omega t + \varphi_2)$ $\underline{i} = I e^{j\varphi_2}$

avec $\underline{u} = Z_{eq} \cdot \underline{i}$ et $Z_{eq} = |Z| e^{j\varphi}$

$\underline{u} = Z_{eq} \underline{i} \Rightarrow$ u et i en phase si Z_{eq} est un réel $\Rightarrow \varphi = 0$

$$Z_{eq} = \frac{(R + jL\omega)\left(R + \frac{1}{jC\omega}\right)}{R + jL\omega + R + \frac{1}{jC\omega}} = \frac{(R + jL\omega)(1 + jRC\omega)}{1 + 2jRC\omega - LC\omega^2} = \frac{\overbrace{R(1 - LC\omega^2) + j(L\omega + R^2C\omega)}^{N}}{\underbrace{1 - LC\omega^2 + 2jRC\omega}_{D}}$$

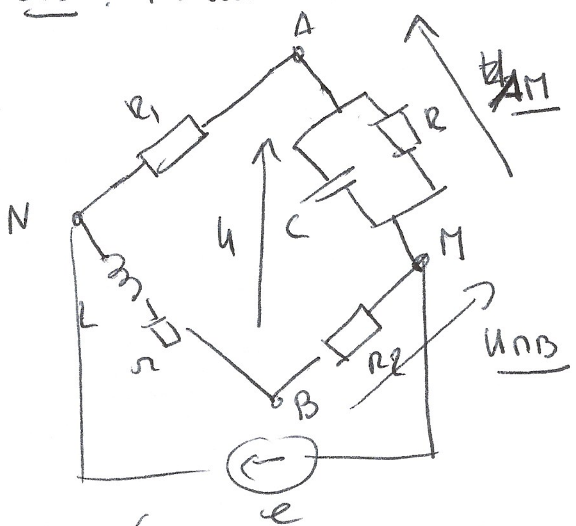
$\varphi = 0$ si $\arg N = \arg(D) \Rightarrow \varphi_N = \varphi_D$

$\operatorname{tg} \varphi_N = \operatorname{tg} \varphi_D \rightarrow \dfrac{L\omega + R^2C\omega}{1 - LC\omega^2} = \dfrac{2RC\omega}{1 - LC\omega^2}$

$$\begin{cases} L\omega + R^2C\omega = 2R^2C\omega \Rightarrow R = \sqrt{\frac{L}{C}} \\ L\omega = R^2C\omega . \end{cases}$$

On a alors $Z_{eq} = \dfrac{R(1-LC\omega^2) + j\,2R^2C\omega}{1-LC\omega^2 + j\,2RC\omega} = R$

ex6 : Mesure d'une inductance.



$\underline{u} = \underline{u_{AM}} + \underline{u_{MB}}$.

$$\underline{u_{AM}} = \frac{Z_{eq}\cdot \underline{e}}{R_1 + Z_{eq}} = \frac{\frac{R}{1+jRC\omega}\,\underline{e}}{R_1 + \frac{R}{1+jRC\omega}} = \frac{R\,.\,\underline{e}}{R+R_1+jRR_1C\omega}$$

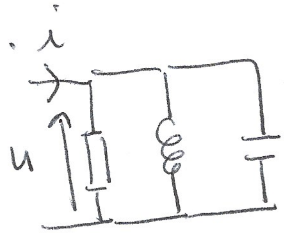
$$\underline{u_{MB}} = \ominus \frac{R_2}{r+R_2+jL\omega}$$

$$\underline{u} = \left(\frac{R}{R+R_1+jRR_1C\omega} - \frac{R_2}{r+R_2+jL\omega}\right)\underline{e} = 0 \Rightarrow \frac{R}{R+R_1+jRR_1C\omega} = \frac{R_2}{r+R_2+jL\omega}$$

$$\Rightarrow R\cdot r + \cancel{R\cdot R_2} + jRL\omega = \cancel{RR_2} + R_1R_2 + j\,RR_1R_2C\omega .$$

$$\begin{cases} R\cdot r = R_1R_2 \Rightarrow \boxed{r = \frac{R_1R_2}{R}} \\ R L\cancel{\omega} = RR_1R_2C\cancel{\omega} \quad \boxed{L = R_1R_2C} \end{cases}$$

ex5. i



$i(t) = i_0\cos\omega t \Rightarrow \underline{i}(t) = i_0 e^{j\omega t} \rightarrow \underline{i} = i_0$

$u(t) = U\cos(\omega t + \varphi) \Rightarrow \underline{u}(t) = \underline{u}\, e^{j\omega t}$ et $\underline{u} = U e^{j\varphi}$.

$\underline{u} = Z_{eq}\cdot\underline{i} \Rightarrow \underline{u} = \dfrac{1}{Y_{eq}}\underline{i}$ et $\underline{Y_{eq}} = \dfrac{1}{R} + j\left(C\omega - \dfrac{1}{L\omega}\right)$

$$\Rightarrow U(\omega) = \frac{i_0}{\sqrt{\frac{1}{R^2} + \left(C\omega - \frac{1}{L\omega}\right)^2}}$$

$U(\omega) \to 0$ pour $\omega \to 0$

$U(\omega) \to \infty$ pour $\omega \to \infty$

$U(\omega)$ max pour $C\omega - \dfrac{1}{L\omega} = 0 \to \omega_0 = \dfrac{1}{\sqrt{LC}}$

$$\underline{u} = \frac{\underline{i}}{\frac{1}{R} + j\left(C\omega - \frac{1}{L\omega}\right)} = \frac{\underline{i}\,.\,R}{1+j\left(RC\omega - \frac{R}{L\omega}\right)} = \frac{\underline{i}\,.\,R}{1+jQ'\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \Rightarrow \begin{cases} RC = \frac{Q'}{\omega_0} \\ \frac{R}{L} = Q'\omega_0 \end{cases} \boxed{Q' = R\sqrt{\frac{C}{L}}}$$