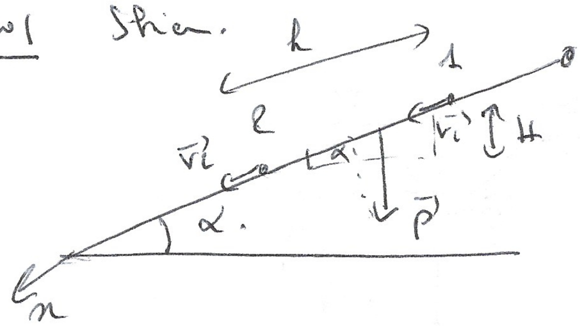
Correction TD energie :

ex1 Skieur.



TEC

$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = W_{1\to 2}(\vec{P})$

$= mg\cancel{H}$

et $\sin\alpha = \frac{H}{h}$ $\Rightarrow$ $= mg\sin\alpha . h$.

$$\boxed{\frac{v_2^2 - v_1^2}{2 . g\sin\alpha} = h.}$$

AN $h = \frac{100-1}{2 . 9{,}81 . \sin 30°} = 10\,m.$

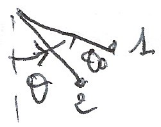
PFD $m\vec{a} = \vec{P} + \vec{R_N}$

proj/x. $\cancel{m}\ddot{x} = \cancel{m}g\sin\alpha \rightarrow \dot{x} = g\sin\alpha . t + cste$. à $t=0$ $v = \dot{x} = v_1$

$\dot{x} = g\sin\alpha t + v_1 \Rightarrow \boxed{\Delta t = \frac{v_2 - v_1}{g\sin\alpha}.}$

AN $\Delta t = \frac{9 . 2}{9{,}81} = \boxed{1{,}8\,s = \Delta t}$

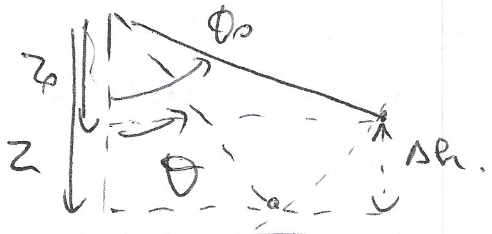
ex2 Pendule simple.



$\theta_0 = 60°$

$\frac{1}{2}\cancel{m}v_2^2 - \underbrace{\frac{1}{2}mv_1^2}_{=0} = \underbrace{W_{1\to 2}(\vec{T})}_{=0} + W_{1\to 2}(\vec{P})$

$= \cancel{m}g\,\Delta h.$



$\Delta h = \ell\cos\theta - \ell\cos\theta_0$.

$= \ell(\cos\theta - \cos\theta_0)$.

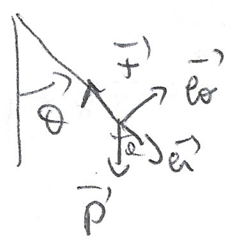
$$\boxed{v_2^2 = v(\theta)^2 = 2\ell g(\cos\theta - \cos\theta_0).}$$

avec $E_p(\text{pesanteur}) = -mgz$.

Par le TEM : $\frac{1}{2}mv_1^2 - mg\ell\cos\theta_0 = \frac{1}{2}m v_2^2 - mg\ell\cos\theta$.

$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mg\ell(\cos\theta - \cos\theta_0) \rightarrow \boxed{v_2^2 = 2\ell g(\cos\theta - \cos\theta_0)}$

2°]



$m\vec{a} = \vec{P} + \vec{T}$

et $\vec{OM} = \ell\vec{e_r}$

$\vec{v} = \ell\dot{\theta}\vec{e_\theta}$

$\vec{a} = -\ell\dot{\theta}^2\vec{e_r} + \ell\ddot{\theta}\vec{e_\theta}$

Projection du PFD suivant $\vec{e_r}$:

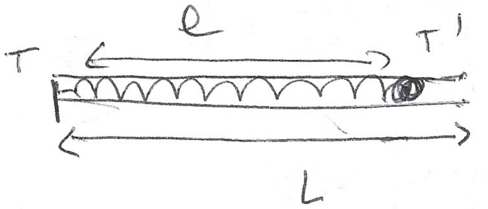
$$-m\ell\dot{\theta}^2 = mg\cos\theta - T$$

$$-m\frac{v^2}{\ell} = mg\cos\theta - T \Rightarrow T = mg\cos\theta + m\frac{v^2}{\ell}$$

$$= mg\cos\theta + \frac{m}{\ell}\cdot 2\ell g\left(\cos\theta - \cos\theta_0\right)$$

$$\boxed{T(\theta) = 3mg\cos\theta - 2mg\cos\theta_0}$$

ex 3 Ressort horizontal.



$\vec{u}$

$L = \ell_0$

à $t=0$ $\ell = \frac{\ell_0}{5}$.

en A $v_A = 0$, $\ell_A = \frac{\ell_0}{5}$ — en B v_B ?.. $\ell_B = \ell_0$

1] Calcul de v_B. a) Par le TEC : $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \int_A^B \vec{T}\cdot d\vec{r}$

avec $\left.\begin{array}{l}\vec{T} = -k(\ell-\ell_0)\vec{u} \\ d\vec{r} = d\ell\,\vec{u}\end{array}\right\}$ $\int_A^B \vec{T}\cdot d\vec{r} = \int_{\frac{\ell_0}{5}}^{\ell_0} -k(\ell-\ell_0)\,d\ell$

$$= \left[-k\frac{(\ell-\ell_0)^2}{2}\right]_{\frac{\ell_0}{5}}^{\ell_0} = \frac{k}{2}\left(\frac{\ell_0}{5}-\ell_0\right)^2$$

$$\frac{1}{2}mv_B^2 = \frac{k}{2}\left(-\frac{4\ell_0}{5}\right)^2 = \frac{8k\ell_0^2}{25} \Rightarrow v_B^2 = \frac{16}{25}\frac{k}{m}\cdot\ell_0^2 \rightarrow \boxed{v_B = \frac{4}{5}\omega_0\ell_0}$$

b] Par le TEM $E_m = \frac{1}{2}mv^2 + mgz + \frac{1}{2}k(\ell-\ell_0)^2 = cste$. $\Rightarrow$

$E_m(A) = E_m(B)$

$\frac{1}{2}mv_A^2 + mgz_A + \frac{1}{2}k\left(\frac{\ell_0}{5}-\ell_0\right)^2 = \frac{1}{2}mv_B^2 + mgz_B + 0$. avec $z_A = z_B$ et $v_A = 0$.

$$\frac{1}{2}mv_B^2 = \frac{1}{2}k\left(\frac{\ell_0}{5}-\ell_0\right)^2 \Rightarrow \boxed{v_B = \frac{4}{5}\omega_0\ell_0}$$

2] $E_m = cste \Rightarrow \frac{dE_m}{dt} = 0$ $\frac{d}{dt}\left[\frac{1}{2}m\dot{\ell}^2 + \frac{1}{2}k(\ell-\ell_0)^2\right] = 0$.

$$\frac{1}{2} m . 2\ddot{\ell}\cdot\dot{\ell} + \frac{1}{2} k . \dot{\ell}(\ell-\ell_0) = 0$$

$$m\ddot{\ell} + k(\ell-\ell_0) = 0 \Rightarrow \ddot{\ell} + \frac{k}{m}\ell = \frac{k}{m}\ell_0 . \qquad \frac{k}{m} = \omega_0^2 .$$

$$\ell(t) = \underbrace{A\cos(\omega_0 t+\varphi)}_{SG_0} + \underbrace{\ell_0}_{S_P} \qquad \text{à } t=0 \quad \ell = \frac{\ell_0}{5} \quad \dot{\ell} = 0.$$

$$\Rightarrow \begin{cases} \frac{\ell_0}{5} = A\cos\varphi + \ell_0 \\ 0 = -A\omega_0\sin\varphi \Rightarrow \varphi = 0. \end{cases}$$

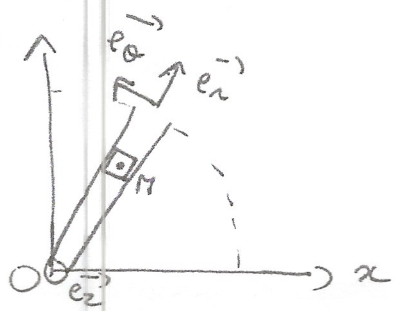
et $A = \frac{\ell_0}{5} - \ell_0 = -\frac{4}{5}\ell_0$.

$$\Rightarrow \ell(t) = -\frac{4}{5}\ell_0\cos\omega_0 t + \ell_0 = \boxed{\frac{4}{5}\ell_0\cos(\omega_0 t+\pi) + \ell_0 = \ell(t).}$$

3] en T' $\ell = \ell_0 \Rightarrow \cos(\omega_0 t + \pi) = 0 \Rightarrow \omega_0 t + \pi = -\frac{\pi}{2} \rightarrow \boxed{t_1 = \frac{\pi}{2\omega_0}}$

Ex 4.

$(Oxyz)$ lié au référentiel terrestre. $(\vec{e_r}, \vec{e_\theta}, \vec{e_z}) \rightarrow$ lié au tube.
en mvt / R_{TL}
$\dot{\theta} = \omega = cste$.



M : cube.

$\vec{OM} = r(t)\vec{e_r}$

$\vec{v}_{M/R_{TL}} = \dot{r}(t)\vec{e_r} + r\omega\vec{e_\theta}$

$\vec{a}_{M/R_{TL}} = (\ddot{r}(t) - r\omega^2)\vec{e_r} + 2\dot{r}\omega\vec{e_\theta}$.

Inventaire des forces s'exerçant ds R_{TL} (sans frottement).

$\vec{P} = -mg\vec{e_z}$ $\quad \vec{T} = -T\vec{e_r}$

$\vec{R}$: réaction du tube sur le cube $\quad \vec{R} = R_z\vec{e_z} + R_\theta . \vec{e_\theta}$.

PFD ds R_{TL} appliqué à M : $m.\vec{a} = \vec{P} + \vec{T} + \vec{R}$

$\vec{e_r}$: $m[\ddot{r} - r\omega^2] = -T$

$/\vec{e_\theta}$ $\quad 2m\dot{r}\omega = R_\theta$.

$/\vec{e_z}$ $\quad 0 = -mg + R_z \rightarrow \boxed{R_z = mg.}$

ex4.

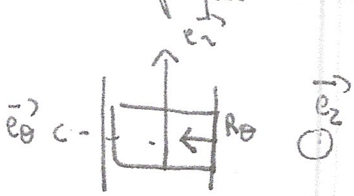
1) Cube immobile (tenu par un fil) par rapport au tube. → $r = \text{cste}$. ($\vec{a}/_{R_{TL}} \neq \vec{0}$).

→ $\boxed{T = r\omega^2}$.

2) On tire le cube vers soi → $R_\theta =$

Cube plaqué suivant $(-\vec{e_\theta})$ ⇒ R_θ dirigé suivant $\vec{e_\theta}$ → $2m\dot{r}\omega > 0$

or $\dot{r} < 0$ → $\omega < 0$ → manège tourne ds le sens rétrograde.



3) On relâche le fil → $T = 0$ → $\ddot{r} = r\omega^2$

$\dot{r} > 0$ et $\omega < 0$ → R_θ dirigé suivant $-\vec{e_\theta}$ → cube plaqué contre la face gauche.

On abandonne le cube ($T=0$)

4) à l'état initial $r = \frac{R}{2}$. jusqu'à l'état final $r = R$.

$v/_{R_n} = 0$; $v_{R_n}/ = ?$

$v/_{R_{TL}} = \frac{R}{2}\omega$; $v_{R_{TL}} = ?$

Forces appliquées ds R_{TL} : $\vec{R}$; $\vec{P}$.

$\ddot{r} = r\omega^2$ → $r(t) = A\,\mathrm{ch}\,\omega t + B\,\mathrm{sh}\,\omega t$. à $t=0$ $\frac{R}{2} = A$

$\dot{r}(t) = A\omega\,\mathrm{sh}\,\omega t + B\omega\,\mathrm{ch}\,\omega t$ $t=0$ $\dot{r} = 0$ → $B = 0$.

$$\boxed{r(t) = \frac{R}{2}\,\mathrm{ch}\,\omega t}$$

Théorème de l'énergie cinétique ds R_{TL} $\Delta E_c = W_{\vec{P}} + W_{\vec{R}}$.

$\vec{dM}/_{R_{TL}} = dr\,\vec{e_r} + r\,d\theta\,\vec{e_\theta}$ ⇒ $W_{\vec{R}} = \int \vec{R}\cdot\vec{dM} = \int R_\theta \cdot r\,d\theta$.

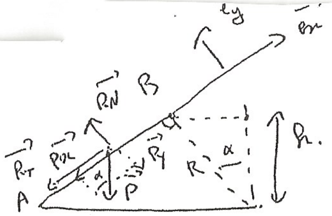
$W_{\vec{R}} = \int 2m\dot{r}\omega \cdot r\omega\,dt = 2m\omega^2 \int r\,\dot{r}\,\frac{dt}{dr} = 2m\omega^2 \int_{\frac{R}{2}}^{R} \frac{r^2}{\cancel{2}}\,dr$

$\Delta E_c = 2m\omega^2 \left|\frac{R^2}{2}\right|_{\frac{R}{2}}^{R} = \cancel{2}m\omega^2\left[\frac{R^2}{\cancel{2}} - \frac{R^2}{4}\right] = \boxed{\frac{3m\omega^2R^2}{4} = \Delta E_c/_{R_{TL}}}$.

Rq $\frac{1}{2}m v_g^2/_{R_{TL}} - \frac{1}{2}m v_i^2/_{R_{TL}} = \frac{3m\omega^2R^2}{4}$ → $v_{g/R_{TL}}^2 = \frac{3}{2}\omega^2R^2 + v_i^2/_{R_{TL}} = \frac{3}{2}\omega^2R^2 + \frac{R^2}{4}\omega^2$

$v_g^2/_{R_{TL}} = \frac{7}{4}\omega^2R^2$ → $\boxed{v_g/_{R_{TL}} = \frac{\sqrt{7}}{2}\omega R.}$

Co 5.



$\cos\alpha = \frac{h}{R} \Rightarrow h = R\cos\alpha.$

et $\operatorname{tg}\alpha = \frac{R}{AB} \Rightarrow \boxed{AB = \frac{R}{\operatorname{tg}\alpha}}$

$\vec{OM} = x(t)\vec{e_x}$

$\vec{v} = \dot{x}(t)\vec{e_x}$

$\vec{a} = \ddot{x}(t)\vec{e_x}$

$\vec{P} = -mg\sin\alpha\,\vec{e_x} - mg\cos\alpha\,\vec{e_y}$

$\vec{R} = R_N\vec{e_y} - R_T\vec{e_x}$ et $R_T = f R_N$.

1] Pas de frottements. $R_T = 0$ et $W_{A\to B}(\vec{R_N}) = 0$

T.E.C $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = W_{A\to B}(\vec{P}) = -mgh = -mgR\cos\alpha.$

a) $\boxed{v_B^2 = v_A^2 - 2gR\cos\alpha.}$

b) $v_B^2 > 0 \Rightarrow \boxed{v_A > \sqrt{2gR\cos\alpha}}$ $\boxed{v_{A\ell}}$ $= \sqrt{2\cdot10\cdot2\cdot\frac{\sqrt{3}}{2}} \simeq \cancel{\sqrt{4\cdot1{,}8\cdot10}} \cancel{\simeq\sqrt{72}}$

AN: $v_{A\ell} = \sqrt{20\cdot1{,}73} \simeq \sqrt{36} \simeq 6\,m\,s^{-1}.$

c] Application du PFD. $m\vec{a} = \vec{P} + \vec{R}.$

proj$/\vec{e_x}$: $m\ddot{x} = -mg\sin\alpha \Rightarrow$ $\boxed{\begin{array}{l}\ddot{x} = -g\sin\alpha\,t + v_A.\\ x(t) = -\frac{g\sin\alpha}{2}t^2 + v_A\cdot t\end{array}}$

le temps de parcours τ est: $\boxed{\tau = \frac{v_A - v_B}{g\sin\alpha}}$

et donc $\boxed{\tau = \frac{v_A - \sqrt{v_A^2 - 2gR\cos\alpha}}{g\sin\alpha}}$

2] Les frottements sont pris en compte. $R_T \neq 0$ et $R_T = f\cdot R_N.$

$m\vec{a} = \vec{P} + \vec{R}$ projeté suivant $\vec{e_y}$ donne: $R_N - mg\cos\alpha = 0 \to R_N = mg\cos\alpha.$

d'où $R_T = f mg\cos\alpha.$

TEC $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = W_{A\to B}(\vec{P}) + W_{A\to B}(R_T)$

$= -mgR\cos\alpha - fmg\cos\alpha\cdot AB = -\left[mgR\cos\alpha + fmg\cos\alpha\frac{R}{\operatorname{tg}\alpha}\right.$

$\boxed{v_B^2 = v_A^2 - 2gR\cos\alpha\left(1 + \frac{f}{\operatorname{tg}\alpha}\right) > 0}$

PFD $\Rightarrow$ $\boxed{v_{A\ell} = \sqrt{2gR\cos\alpha\left(1 + \frac{f}{\operatorname{tg}\alpha}\right)}}$

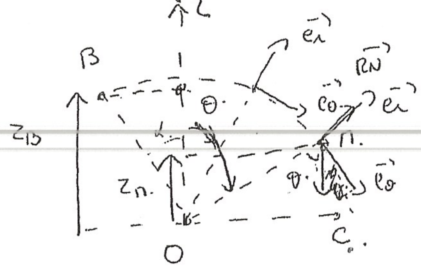
PFD projeté suivant $\vec{e_x}$ $m\ddot{x} = -mg\sin\alpha - fmg\cos\alpha = -mg\sin\alpha\left[1 + \frac{f}{\operatorname{tg}\alpha}\right]$

$\Rightarrow \ddot{x} = -g\sin\alpha\left[1 + \frac{f}{\operatorname{tg}\alpha}\right]$

$x(t) = -g\sin\alpha\left[1 + \frac{f}{\operatorname{tg}\alpha}\right]t + v_A$. τ vérifie.

$\tau = \frac{v_A - v_B}{g\sin\alpha\left[1 + \frac{f}{\operatorname{tg}\alpha}\right]} \Rightarrow \boxed{\tau = \frac{v_A - \sqrt{v_A^2 - 2gR\cos\alpha\left(1 + \frac{f}{\operatorname{tg}\alpha}\right)}}{g\sin\alpha\left[1 + \frac{f}{\operatorname{tg}\alpha}\right]}}$

3] Pas de frottements.



$\vec{OM} = R\vec{e_r}$

$\vec{v} = R\dot{\theta}\vec{e_\theta} = v\vec{e_\theta}$

$\vec{a} = R\ddot{\theta}\vec{e_\theta} - R\dot{\theta}^2\vec{e_r}$

$\vec{P} = m\vec{g} = -mg\cos\theta\,\vec{e_r} + mg\sin\theta\,\vec{e_\theta}$

$\vec{R} = R_N\vec{e_r}$

$m\vec{a} = \vec{P} + \vec{R}$ en proj$/\vec{e_r}$

$-mR\dot{\theta}^2 = R_N - mg\cos\theta.$

$\Rightarrow R_N = mg\cos\theta - m\frac{v^2}{R}.$ (car $v = R\dot{\theta}$)

Calcul de $v(M)$ en utilisant le TEC entre B et M.

$\frac{1}{2}mv^2 - \frac{1}{2}mv_B^2 = W_{P\,B\to M} = \int_{z_B}^{z_M} -mg\,dz = -mg(z_M - z_B).$

avec $z_M = R\cos\theta.$ et $z_B = R\cos\alpha.$

d'où $v^2 - v_B^2 = 2g(R\cos\alpha - R\cos\theta) \Rightarrow v^2 = v_B^2 + 2gR(\cos\alpha - \cos\theta)$

$= v_A^2 - 2gR\cos\alpha + 2gR\cos\alpha - 2gR\cos\theta.$

$\boxed{v^2 = v_A^2 - 2gR\cos\theta.}$ Rq Résultat obtenu immédiatement en appliquant le TEC entre A et M.

$R_N = mg\cos\theta - \frac{mv_A^2}{R} + 2mg\cos\theta = \boxed{3mg\cos\theta - \frac{mv_A^2}{R} = R_N.}$

Il n'y a pas décollage avant le sommet si $R_N > 0$ pr $\cancel{\theta < \theta}$ $2\pi - \alpha < \theta < 2\pi.$

$R_N > 0 \Rightarrow v_A^2 < 3\cancel{m}gR\cos\theta.$

pr $\theta = 2\pi - \alpha \Rightarrow v_A^2 < 3gR\cos\alpha.$

$\theta = 2\pi$ $v_A^2 < 3gR.$

donc $\boxed{v_A < \sqrt{3gR\cos\alpha}.}$

Le palet quitte la piste lorsque $R_N = 0$,

$\boxed{\cos\theta_d = \frac{v_A^2}{3gR}}$