Correction DS3 . 25-25

Optique

Ex1 :

I.A 1°) Modèle simplifié de l'œil.

$n \sin i = \sin r$.

2°)

• Triangle CIH : $\cos i = \dfrac{\overline{CH}}{R_c} \Rightarrow \boxed{\overline{CH} = R_c \cdot \cos i}$.

$\sin i = \dfrac{\overline{HI}}{R_c} \rightarrow \boxed{\overline{HI} = R_c \cdot \sin i}$.

$tg(r-i) = \dfrac{\overline{HI}}{\overline{HA_i}}$ ds le triangle (HIA_i) $\Rightarrow \overline{HA_i} = \dfrac{\overline{HI}}{tg(r-i)} = \boxed{\dfrac{R_c \cdot \sin i}{tg(r-i)} = \overline{HA_i}}$

3°) $\overline{CA_i} = \overline{CH} + \overline{HA_i} = \boxed{R_c \cdot \cos i + \dfrac{R_c \cdot \sin i}{tg(r-i)} = \overline{CA_i}}$

4°) a) $\sin i = \dfrac{\overline{HI}}{R_c} = \dfrac{h}{R_c} \ll 1 \Rightarrow$ $\sin i \simeq i$ et $\cos i \simeq 1$.

$n \sin i = \sin r$ devient $n . i = r$.

d'où $\overline{CA_i} = R_c + R_c \cdot \dfrac{i}{r-i} = R_c + R_c \cdot \dfrac{i}{(n-1)\, i} = R_c \left(1 + \dfrac{1}{n-1}\right) = \boxed{\dfrac{n R_c}{n-1} = \overline{CA_i}}$

b) $H = S$ $\quad f_i = \overline{SF_i} = \overline{HA_i} = \dfrac{R_c \cdot i}{(n-1)\, i} = \boxed{\dfrac{R_c}{n-1} = f_i}$

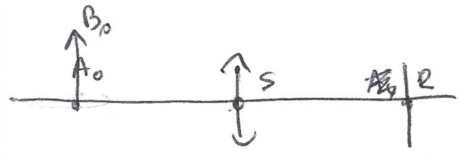
c) $V = 60\delta = \dfrac{1}{f_i} \Rightarrow f_i = \dfrac{1}{60}\,m = \boxed{1{,}67\,cm = f_i}$ $\quad \boxed{R_c = f_i(n-1) = 5{,}5\,mm}$

avec $n = 1{,}33$.

I-B Modèle simplifié de l'œil pour la vision de près.

I.6.

a



$\overline{SR} = 1,67\,cm = cte.$

$V = \dfrac{1}{\overline{SF_i}}$ $\quad d_{min} = 25\,cm.$

$p_0 = \overline{SA_0} < 0$

$p_i = \overline{SA_i} > 0.$

$\dfrac{1}{\overline{SA_i}} - \dfrac{1}{\overline{SA_0}} = \dfrac{1}{f'}$ Relation de Descartes.

$\boxed{\dfrac{1}{p_i} - \dfrac{1}{p_0} = V}$ $\quad [V] = L^{-1}$ $\quad$ unité dioptrie (1δ)

$1\,\delta = 1\,m^{-1}.$

b] $\overline{SA_0} = -d_{min} = p_0$

$p_i = \overline{SR}$

$\boxed{V_{max} = \left[\dfrac{p_0 \cdot p_i}{p_0 - p_i}\right]^{-1}}$ (objet au PP)

AN $\quad V_{max} = \dfrac{[(-25) - 1,67] \cdot 10^{-2}}{(-25) \cdot (1,67) \cdot 10^{-4}} = 63,8\,\delta$ $\quad \boxed{V_{max} = 64\,\delta}$

c] objet au P.R $\quad \overline{SA_0} \to \infty.$ $\quad \boxed{V_{min} = \dfrac{1}{\overline{SR}}}$ AN $\quad V_{min} = \dfrac{1}{1,67 \cdot 10^{-2}} \to \boxed{V_{min} = 60\,\delta}$

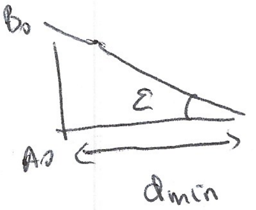
d) $\boxed{A = V_{max} - V_{min} = 4\,\delta}$

I.7] V'_{max} pour $d_m = 35\,cm.$

a)

$A = \underbrace{\left(\dfrac{1}{\overline{SR}} - \dfrac{1}{(-d_{min})}\right)}_{V_{max}} - \underbrace{\left(\dfrac{1}{\overline{SR}}\right)}_{V_{min}} = \dfrac{1}{d_m} = \dfrac{1}{35 \cdot 10^{-2}} = \boxed{2,86\,\delta = A_{presbyte}}$

b)



$\varepsilon = 4 \cdot 10^{-4}\,rad$

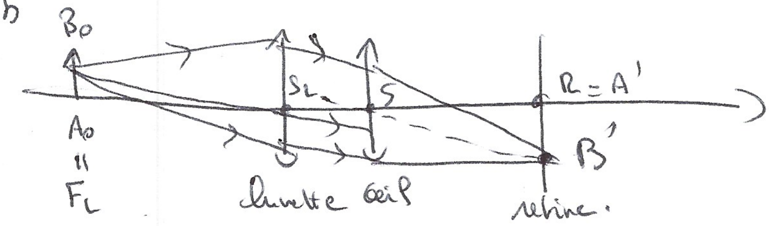
$A_0B_0 = \varepsilon \cdot d_{min} = (4 \cdot 10^{-4} \cdot 35)\,cm = 0,14\,mm.$

c) $d_{min} = 1\,m$ $\quad A_0B_0 = 4 \cdot 10^{-4}\,m = 0,4\,mm.$

La lecture devient difficile.

I-3] Correction.

a,b



$\overline{S_LS} = 2cm.$

$\overline{A_oS} = 25cm.$

$$A_oB_o \xrightarrow[\text{lunette}]{} A_iB_i \xrightarrow[\text{oeil}]{} R = A'B'$$

$A_i = \infty$. oeil qui n'accomode pas.

A_o doit être au foyer objet de la lunette.

$\overline{A_oS_L} = 23cm = f'_L \rightarrow V_L = \frac{1}{23.10^{-2}} = \boxed{4{,}3\delta = V_L}$

c] $A_o \xrightarrow{L_L} A_1 \xrightarrow[\text{accomodant}]{\text{oeil}} R.$

$A_o = ?$

L'oeil accomodant au maximum.

$\overline{SA_1} = -1m \Rightarrow \overline{S_LA_1} = \overline{S_LS} + \overline{SA_1}$

$= 2 - 1 = -98 cm.$

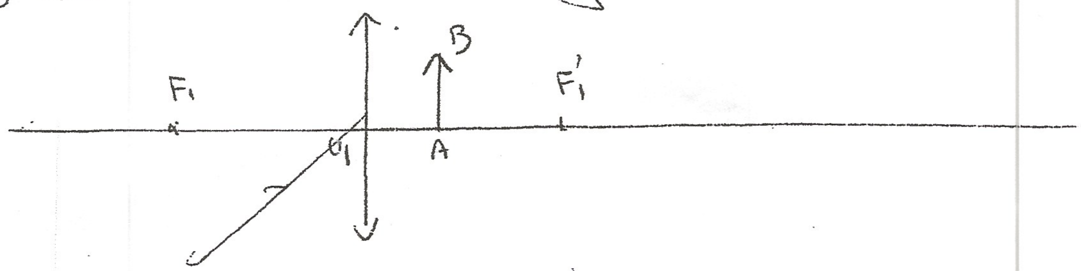
et $f'_L = 23cm.$

$$\frac{1}{\overline{S_LA_1}} - \frac{1}{\overline{S_LA_o}} = \frac{1}{f'_L} \Rightarrow \frac{1}{\overline{S_LA_o}} = \frac{1}{\overline{S_LA_1}} - \frac{1}{f'_o} \rightarrow \overline{S_LA_o} = \frac{\overline{S_LA_1} \cdot f'_o}{f'_o - \overline{S_LA_1}} = \frac{23\cdot(-98)}{23+98}$$

$\boxed{\overline{S_LA_o} = -18{,}6 cm.}$

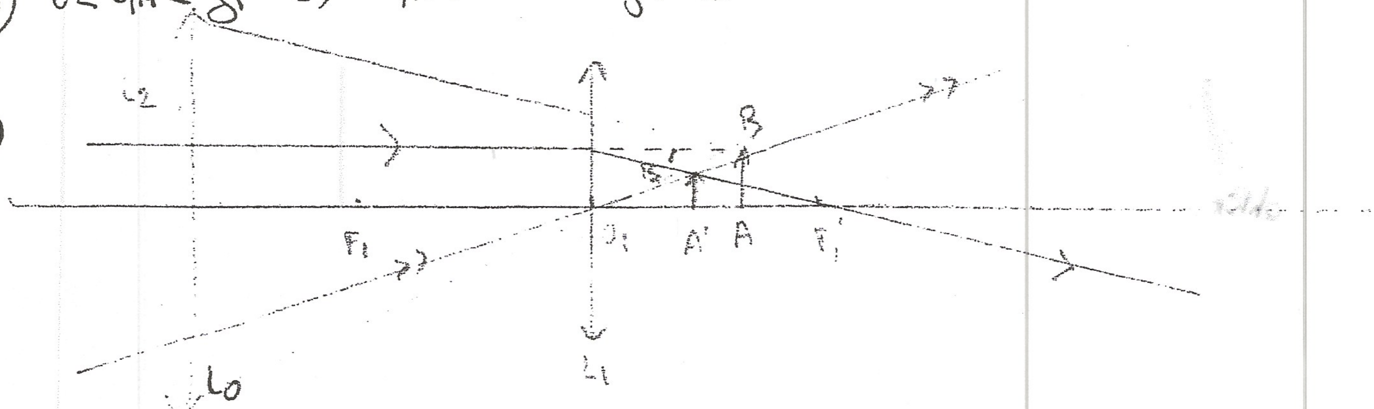
DZ

I] Étude de la lentille convergente L_1.



1) $0 < \overline{O_1A} < f_1'$ => $\overline{O_1A} > 0$ l'objet est virtuel.

2)



3) L'image est réelle. Pour obtenir un objet AB virtuelle par la lentille L_1, on peut utiliser un objet réel A_0B_0 dont on forme une image réelle (A_1B_1) à l'aide L_0, puis interposer devant le faisceau émergent, la lentille L_1.

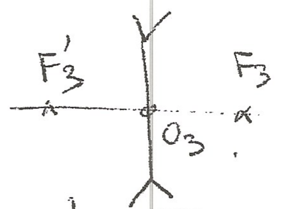
$$\frac{1}{\overline{O_1A'}} = \frac{1}{\overline{O_1A}} + \frac{1}{f_1'} \rightarrow \overline{O_1A'} = \frac{f_1' \cdot \overline{O_1A}}{f_1' + \overline{O_1A}} = \frac{10 \cdot 15}{10+15} = \frac{10 \cdot 15}{25}$$

$$= \frac{10 \cdot 3}{5} = \frac{30}{5} = \boxed{6\,cm = \overline{O_1A'}}$$

1/ Principe du téléobjectif : L_1 convergente + L_3 divergente

$$AB \xrightarrow{(L_1)} A_1B_1 \xrightarrow{L_3} \underset{\text{sur } P}{A'B'}$$

1] $A'B'$ à l'∞ $\Rightarrow$ $A_1 = F_3$; AB à l'∞ $\Rightarrow$ $A_1 = F'_1$ $\}$ d'où $F_3 = F'_1$.



d'où $\overline{O_1O_3} = \overline{O_1F'_1} + \underbrace{\overline{F'_1F_3}}_{=0} + \overline{F_3O_3}$ $\boxed{f'_1 + f'_3 = \overline{O_1O_3}}$

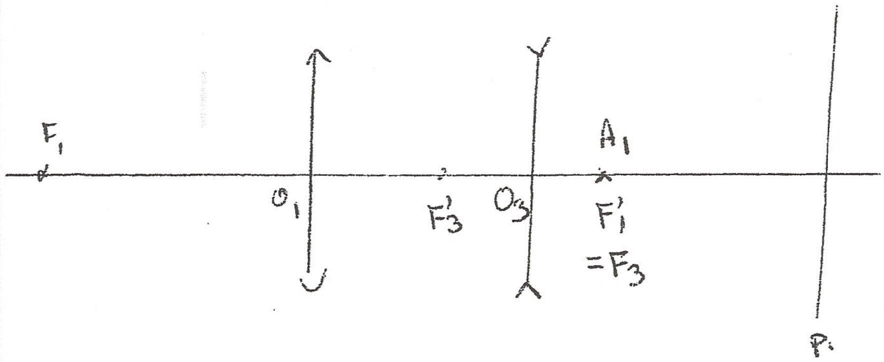
$f'_3 = \overline{O_3F'_3} < 0$

$f_3 = \overline{O_3F_3} > 0$

$= -f'_3$

$\Rightarrow \overline{F_3O_3} = f$

AN $\left.\begin{array}{l} f'_1 = 10\,cm. \\ f'_3 = -3\,cm. \end{array}\right\}$ $\overline{O_1O_3} = 7\,cm.$



- A_1B_1 doit être virtuel donc situé après O_3 car une lentille divergente donne une image virtuelle d'un objet réel.

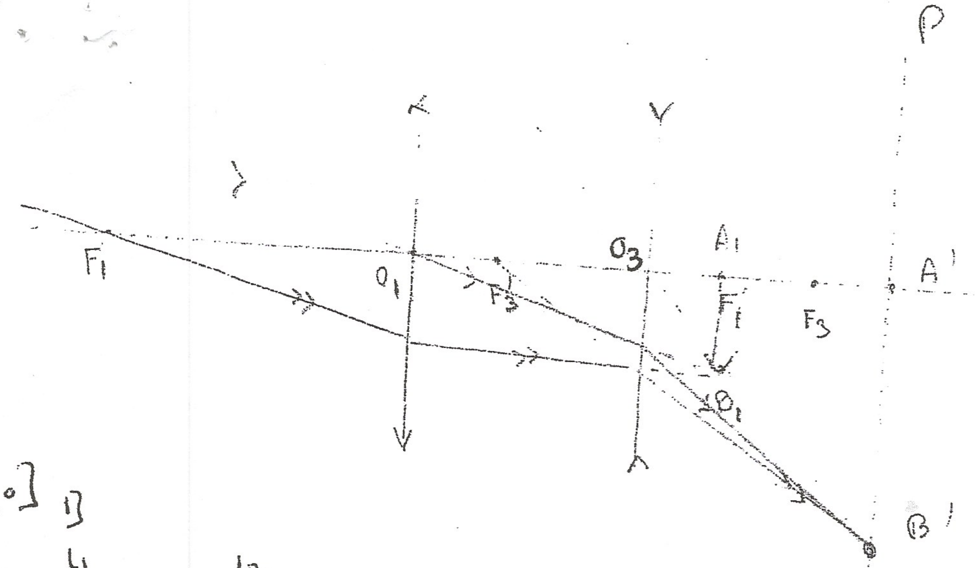
- Si on écarte les 2 lentilles A_1 est situé avant F_3 $\overline{O_3A_1} < \overline{O_3F_3}$

Si on rapproche " A_1 " après F_3 $\overline{O_3A_1} > \overline{O_3F_3}$

et on veut $\overline{O_3A'} > 0$ or $\frac{1}{\overline{O_3A'}} = \frac{1}{\overline{O_3A_1}} + \frac{1}{f'_3} > 0 \Rightarrow$

$\frac{1}{\overline{O_3A}} > -\frac{1}{f'_3} \rightarrow \overline{O_3A_1} < -f'_3 \Rightarrow \boxed{\overline{O_3A_1} < \overline{O_3F_3}}$ On doit donc écarter les 2 lentilles

(4)



4°] B

$$AB \xrightarrow{L_1} A_1B_1 \xrightarrow{L_3} A'B' \qquad \text{avec } A_1 = F_1'$$

$$\boxed{\overline{F_3F_1'} \cdot \overline{F_3'A'} = -f_3'^2}$$

avec $\boxed{\overline{F_3'A'} = \overline{F_3'O_3} + \overline{O_3A'}}$

AN $\overline{F_3'A'} = -f_3' + \overline{O_3P} = 3 + 10\ \text{cm} = 13\ \text{cm}.$

$$\overline{F_3F_1'} = -\frac{f_3'^2}{\overline{F_3'A'}} = -\frac{9}{13} = \boxed{-0{,}7\ \text{cm} = \overline{F_3F_1'}}$$

4.2]. $\overline{A_1B_1} = f_1'\alpha$ et $\dfrac{\overline{A_3B_3}}{\overline{A_1B_1}} = \dfrac{\overline{O_3A'}}{\overline{O_3A_1}} = \gamma_3 \Rightarrow \boxed{\overline{A_3B_3} = \gamma_3 \cdot f_1' \cdot \alpha.}$

AN $\overline{O_3A'} = \cancel{13}\ 10\ \text{cm}.$

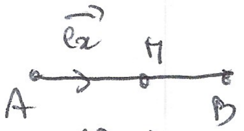
$\overline{O_3A_1} = \overline{O_3F_3'} + \overline{F_3F_1'} = 3 - 0{,}7 = 2{,}3\ \text{cm}. \Rightarrow \left|\overline{A_3B_3}\right| = \dfrac{10}{2{,}3} \cdot 10 \cdot 10^{-1}$

$\overline{O_3F_1'}$

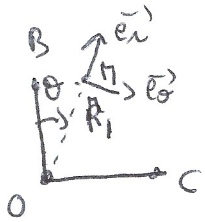
$$\boxed{\left|\overline{A_3B_3}\right| = 4{,}3\ \text{cm}.}$$

Mecanique

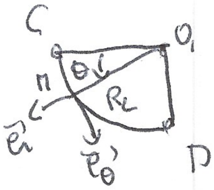
B Application:



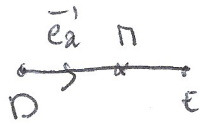
AB = L $\qquad \vec{AM} = x\vec{e_x}$



$\vec{OM} = R_1\vec{e_r}$



$\vec{O_1M} = R_2\vec{e_r}$



DE = H $\qquad \vec{DM} = x\vec{e_x}$

Ma : Mobile en mvt uniforme $\quad v_a = 25\ ms^{-1}$.

1°] $\tau = \frac{AB+BC+CD+DE}{v_0} = \frac{425}{25} = 17\,s$ (1C)

2°] Partie BC $\quad \vec{OM} = R_1\vec{e_r} \quad \vec{v} = R_1\dot\theta\,\vec{e_\theta} \quad \vec{a_1} = -R_1\dot\theta^2\vec{e_r}$ avec $\dot\theta = \frac{v_A}{R_1}$

$\vec{a_1} = -R_1 \cdot \frac{v_A^2}{R_1^2}\vec{e_r} = -\frac{v_A^2}{R_1}\vec{e_r}$ et $BC = \frac{2\pi R_1}{4} = \frac{\pi R_1}{2} \to R_1 = \frac{2\cdot BC}{\pi}$

d'où $\boxed{\vec{a_1} = -\frac{v_A^2\cdot\pi}{2\cdot BC}\vec{e_r}}$ AN $\boxed{a_1 = 0,2\ ms^{-2}}$ (2B)

3°] $\vec{a_2} = -\frac{v_A^2\cdot\pi}{2\cdot CD}$ et $CD = \frac{3}{2}BC \Rightarrow \boxed{a_2 = \frac{2}{3}a_1}$ (3C).

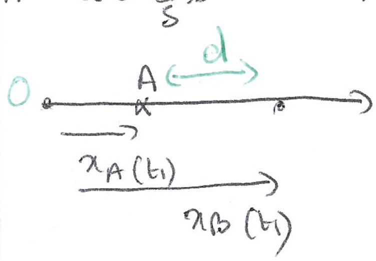
4°] $x_A = v_A \cdot t$ $\qquad x_A = x_B \to t_R = \frac{H}{v_A+v_B} = \frac{200}{25+50} = \frac{8\cdot 25}{3\cdot 25} = \boxed{2,7s = t_R}$

$x_B = -v_B\cdot t + H$

(4B)

5°] $x_A = t_R \cdot v_a = 25\cdot\frac{8}{3} = \boxed{67\ cm = x_A}$ (5B)

6°] $x_A \quad t_0 = \frac{1}{5}s \qquad t_1 = t_R - t_0$



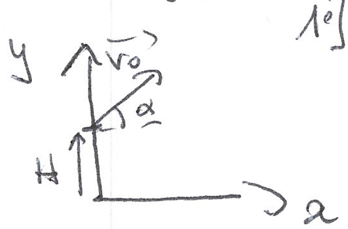
$d = [x_B - x_A](t_1)$

$= -v_B(t_R - t_0) + H - v_A(t_R - t_0)$

et $-v_B t_R + H = +v_a t_R$

$d = v_B t_0 + v_A t_0 = 75\cdot\frac{1}{5}\ cm = 15\ cm$ (7).

Exercice 2.



$m\vec{a} = m\vec{g}$ (1)

1°] $\vec{OM} = x\vec{e_x} + y\vec{e_y} \to \vec{a} = \ddot{x}\vec{e_x} + \ddot{y}\vec{e_y}$.

projection de (1) sur

$\vec{e_x}$: $m\ddot{x} = 0$

$\vec{e_y}$: $m\ddot{y} = -mg$

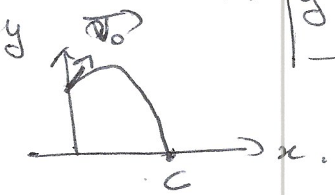
à $t=0$ $\dot{x} = V_0\cos\alpha$, $x = 0$; $\dot{y} = V_0\sin\alpha$, $y = H$.

d'où : $\ddot{x} = 0 \to \dot{x} = V_0\cos\alpha \to \boxed{x(t) = V_0\cos\alpha\, t}$

$\ddot{y} = -g \to \dot{y} = -gt + V_0\sin\alpha$,

$\to \boxed{y(t) = -\frac{g}{2}t^2 + V_0\sin\alpha\, t + H}$

d'où $\boxed{y(x) = -\frac{g}{2}\frac{x^2}{V_0^2\cos^2\alpha} + \frac{\sin\alpha\, x}{\cos\alpha} + H.}$



2°]

$y(x) = -\frac{1}{\cos^2\alpha}\left[\frac{g}{2V_0^2}x^2 - \sin\alpha\cos\alpha\, x - H\cdot\cos^2\alpha\right]$

en C $y_C = 0 \Rightarrow \boxed{\frac{g}{2V_0^2}x_C^2 - \sin\alpha\cos\alpha\, x_C - H\cos^2\alpha = 0.}$ (1)

d'où $\begin{cases} B = -\sin\alpha\cos\alpha \\ C = -H\cos^2\alpha. \end{cases}$

$\left[\frac{g}{2V_0^2}\right] = \frac{LT^{-2}}{L^2T^{-2}} = L^{-1} \Rightarrow \left[\frac{g}{2V_0^2}x_C^2\right] = L$

d'où [B] doit être sans dimension or $[\sin\alpha\cos\alpha] = 1$

[C] " " homogène à une longueur. $[H\cos^2\alpha] = L$.

3°] En dérivant (1) par rapport à α :

$$\frac{g}{2V_0^2}x_C\cdot\underbrace{\frac{dx_C}{d\alpha}}_{0} - \frac{1}{2}\underbrace{\frac{d}{d\alpha}(\sin 2\alpha)}_{2\cos 2\alpha}\cdot x_C - \sin\alpha\cos\alpha\underbrace{\frac{dx_C}{d\alpha}}_{0} + 2H\cos\alpha\sin\alpha = 0$$

d'où $-\cos 2\alpha\cdot x_C + 2H\cos\alpha\sin\alpha = 0 \to \boxed{x_C = \frac{2H\cos\alpha\cdot\sin\alpha}{\cos 2\alpha} = \frac{2H\cos\alpha\sin\alpha}{\cos^2\alpha - \sin^2\alpha}.}$

4°] x_C vérifie (1)

$$\frac{g}{2V_0^2}\cdot\frac{4H^2\cos^2\alpha\sin^2\alpha}{(\cos^2\alpha - \sin^2\alpha)^2} - \frac{2H\cos^2\alpha\sin^2\alpha}{\cos^2\alpha - \sin^2\alpha} - H\cos^2\alpha = 0.$$

$$\frac{2gH^2}{V_0^2}\cos^2\alpha\sin^2\alpha - 2H\cos^2\alpha\sin^2\alpha(\cos^2\alpha - \sin^2\alpha) - H\cos^2\alpha(\cos^2\alpha - \sin^2\alpha)^2$$

$$aH\cos^2\alpha\sin^2\alpha - \cancel{2\cos^4\alpha\sin^2\alpha} + \cancel{2\cos^2\alpha\sin^4\alpha} - \cos^2\alpha\left(\cos^4\alpha + \sin^4\alpha - 2\cos^2\alpha\sin^2\alpha\right)$$

$$- \cos^6\alpha - \cancel{\cos^2\sin^4\alpha} + \cancel{2\cos^4\alpha\sin^2\alpha}.$$

$$aH\cos^2\sin^2\alpha + \cos^2\alpha\sin^4\alpha - \cos^6\alpha = 0.$$

$$aH\sin^2\alpha + \sin^4\alpha - \cos^4\alpha = 0.$$

$$aH\sin^2\alpha + \underbrace{(\sin^2\alpha + \cos^2\alpha)}(\sin^2\alpha - \cos^2\alpha) = 0 \Rightarrow (1+aH)\sin^2\alpha = \cos^2\alpha.$$

$$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \boxed{\frac{1}{1+aH} = \tan^2\alpha.} \quad \text{et} \quad \boxed{a = \frac{2g}{v_0^2}}.$$

5°]

$$x_{cn} = \frac{2H\frac{\sin\alpha}{\cos\alpha}}{1-\tan^2\alpha} = \frac{2H\tan\alpha}{1-\tan^2\alpha} = \frac{2H}{\sqrt{1+aH}} \cdot \frac{1}{1-\frac{1}{aH+1}}.$$

$$= \frac{2H}{\sqrt{1+aH}} \cdot \frac{1+aH}{aH} = \frac{2}{a}\cdot\sqrt{1+aH} = \frac{2}{a}\sqrt{1+\frac{2gH}{v_0^2}}.$$

$$= \frac{2}{av_0}\sqrt{v_0^2+2gH} = \frac{2}{\frac{2g}{v_0^2}\cdot v_0}\sqrt{v_0^2+2gH} \rightarrow \boxed{x_{cn} = \frac{v_0}{g}\sqrt{v_0^2+2gH}.}$$

AN $x_{cn} = 11{,}8$ m.