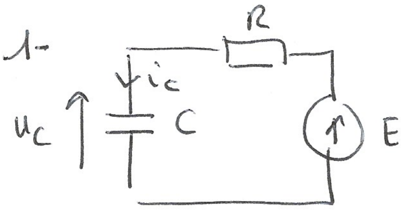
Ex 3 Charge d'un condensateur.

1-



1) $\begin{cases} E = Ri_c + u_c \\ i_c = C\dfrac{du_c}{dt} \end{cases} \rightarrow E = RC\dfrac{du_c}{dt} + u_c$.

$\dfrac{du_c}{dt} + \dfrac{u_c}{\tau} = \dfrac{E}{RC}$ avec $\tau = RC$.

$\left[\dfrac{du_c}{dt}\right] = \dfrac{[u_c]}{T} \Rightarrow [\tau] = T$ vérification $u = Ri \Rightarrow [R] = \dfrac{\text{Tension}}{I}$

$i = C\dfrac{du}{dt} \Rightarrow [C] = \dfrac{I \cdot T}{[\text{Tension}]}$

d'où $[RC]$ est bien homogène à 1 temps.

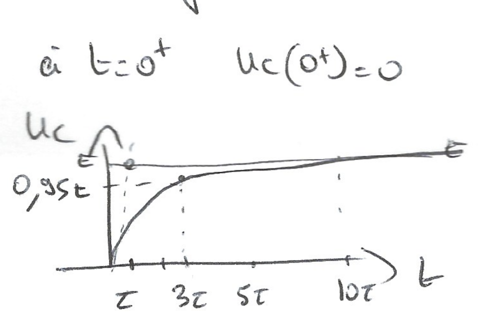
$\boxed{t > 10\tau \Rightarrow u_c \rightarrow \text{régime permanent} = S_p}$

2°) $u_c(0^-) = 0$ car le condensateur est déchargé.
La tension aux bornes du condensateur est continue $\Rightarrow u_c(0^+) = u_c(0^-)$
d'où $u_c(0^+) = 0$.

3) $u_c(t) = Ae^{-\frac{t}{\tau}} + \underset{\text{cte}}{S_p}$ et S_p vérifie $\underbrace{\dfrac{dS_p}{dt}}_{0} + \dfrac{S_p}{\tau} = \dfrac{E}{RC}$.

$\Rightarrow S_p = E$.

4°) à $t = 0^+$ $u_c(0^+) = 0$ $\quad 0 = Ae^{-\frac{0}{\tau}} + E \rightarrow A = -E \Rightarrow \boxed{u_c(t) = E(1 - \exp^{-\frac{t}{\tau}})}$

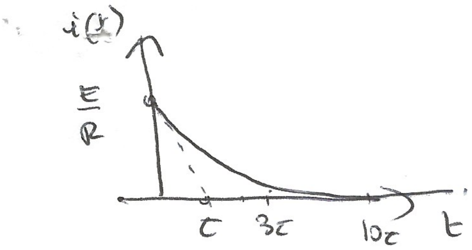


A la fin de la charge $u_c(t > 10\tau) = E$.

5°) $E_{stockée}(t > 10\tau) = \dfrac{1}{2}C\,u_c^2(t > 10\tau) = \dfrac{1}{2}CE^2$.

6°) $i_c(t) = C\dfrac{du_c}{dt} = C \cdot E\left(0 + \dfrac{1}{\tau}\exp^{-\frac{t}{\tau}}\right) = \dfrac{E}{R}\exp^{-\frac{t}{\tau}}$

$i_c(t > 10\tau) \rightarrow 0$.



7°) $P_{Generateur} = E \cdot i_c(t)$ (>0 car convention générateur)

$$= \frac{dE_{Generateur}}{dt}$$

$$dE_{generateur} = E \cdot i_c(t) \cdot dt$$

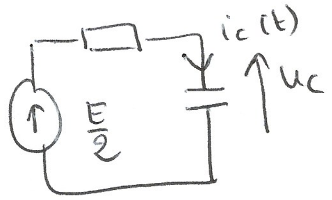
$$E_{fournie} = E_{Generateur}(t > 10\tau) - E_{Generateur}(t=0) = \int_0^{10\tau} \frac{E^2}{R} \exp^{-\frac{t}{\tau}} dt$$

$$= \frac{E^2}{R} \cdot (-\tau) \left[\exp^{-\frac{t}{\tau}}\right]_0^{10\tau} = -\frac{E^2}{R} \cdot RC \left(\underbrace{\exp^{-\frac{10\tau}{\tau}}}_{0} - 1\right) = CE^2$$

8°) $\boxed{\omega = 50\%}$

Second procédé de charge

9°)



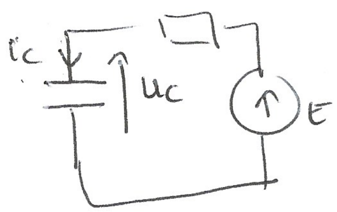
De façon analogue

$$u_c(t) = \frac{E}{2}\left(1 - \exp^{-\frac{t}{\tau}}\right) \quad \text{avec } \tau = RC$$

10°) $u_c(t_1) = \frac{E}{2}\left(1 - \exp^{-\frac{t_1}{\tau}}\right) = 0{,}99 \cdot \frac{E}{2}$. $\quad 1 - 0{,}99 = \exp^{-\frac{t_1}{\tau}}$

$$0{,}01 = 10^{-2} = \exp^{-\frac{t_1}{\tau}} \rightarrow t_1 = -\tau \ln 10^{-2} = (2\ln 10) \cdot \tau = \boxed{t_1 = 4{,}6 \cdot \tau}$$

11°) $u_c(t_1^-) = \frac{E}{2}$ par continuité de u_c $\quad u_c(t_1^+) = \frac{E}{2}$.

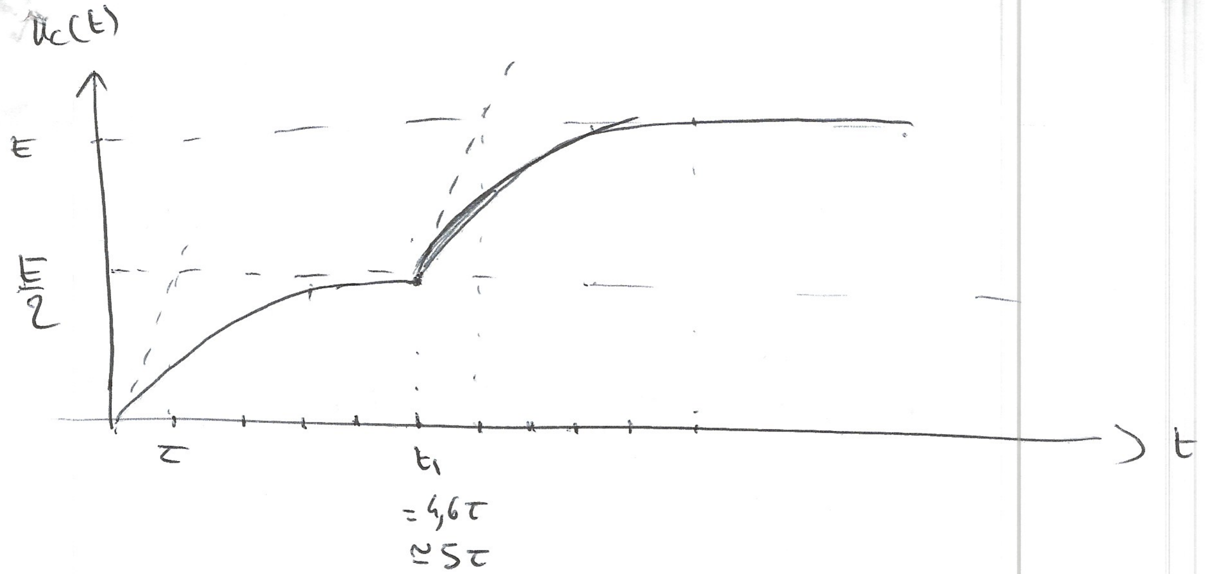


$$\frac{du_c}{dt} + \frac{u_c}{\tau} = \frac{E}{RC}$$

$$u_c(t) = Ae^{-\frac{t}{\tau}} + E \quad \text{et à } t_1^+ \quad u_c(t_1^+) = \frac{E}{2} = A\exp^{-\frac{t_1}{\tau}} + E$$

$$A = -\frac{E}{2}\exp^{\frac{t_1}{\tau}} \Rightarrow u_c(t) = E - \frac{E}{2}\exp^{-\frac{(t-t_1)}{\tau}} = \boxed{E\left(1 - \frac{1}{2}\exp^{-\frac{(t-t_1)}{\tau}}\right) = u_c(t)}$$

12)



13)

1° phase $\quad 0 < t < t_1 \quad u_C(t) = \frac{E}{2}\left(1 - \exp^{-\frac{t}{\tau}}\right)$

$$\Rightarrow i_C(t) = C\frac{du_C}{dt} = C\cdot\frac{E}{2}\cdot\frac{1}{\tau}\exp^{-\frac{t}{\tau}} = \boxed{\frac{E}{2R}\exp^{-\frac{t}{\tau}} = i_C(t) \quad t < t_1.}$$

2° phase $\quad t_1 < t < \infty \quad u_C(t) = E\left(1 - \frac{1}{2}\exp^{-\left(\frac{t-t_1}{\tau}\right)}\right)$

$$i_C(t) = C\frac{du_C}{dt} = CE\cdot\left(-\frac{1}{2}\right)\left(-\frac{1}{\tau}\right)\exp^{-\left(\frac{t-t_1}{\tau}\right)}.$$

$$\boxed{i_C(t) = \frac{E}{2R}\exp^{-\left(\frac{t-t_1}{\tau}\right)}}$$

14) 1° phase.

$$E_{fournie} = \int_0^{t_1}\frac{E}{2}\cdot i_C(t)\,dt = \frac{E^2}{4R}\int_0^{t_1}\exp^{-\frac{t}{\tau}} = \frac{E^2}{4R}\cdot(-\tau)\left[\exp^{-\frac{t}{\tau}}\right]_0^{t_1} = -\frac{CE^2}{4}\left[\underbrace{\exp^{-\frac{t_1}{\tau}}}_{\simeq 0} - 1\right]$$

$$= \frac{CE^2}{4}.$$

2° phase $\quad E_{fournie} = \int_{t_1}^{10\tau} E\cdot i_C(t) = \frac{E^2}{2R}\int_{t_1}^{10\tau}\exp^{-\frac{(t-t_1)}{\tau}}dt = \frac{E^2}{2R}(-\tau)\left[\exp^{-\frac{(t-t_1)}{\tau}}\right]_{t_1}^{10\tau}$

$$= -\frac{CE^2}{2}\left[\underbrace{\exp^{-\frac{(10\tau - t_1)}{\tau}}}_{\simeq 0} - \underbrace{\exp^{0}}_{1}\right] = \frac{CE^2}{2}$$

$E_{stockée} = \frac{1}{2}CE^2$

$$\Rightarrow (E_{fournie})_{totale} = \frac{CE^2}{4} + \frac{CE^2}{2} = \boxed{\frac{3}{4}CE^2 = E_{fournie}} \qquad \Rightarrow \boxed{\omega = \frac{2}{3} = 0,66\%}$$