Correction DS n°2

Exercice 1

$$(3x-4)(x^2-5x+4) \geqslant 0$$

$(3x-4)$: rac $\frac{4}{3}$; (x^2-5x+4) : rac 1 et 4.

x	$-\infty$		1		$4/3$		4		$+\infty$
$3x-4$		$-$			0		$+$		
x^2-5x+4		$+$	0		$-$		0	$+$	
$(3x-4)(x^2-5x+4)$		$-$	0	$+$	0	$-$	0	$+$	

car $a = 1 > 0$

$$\boxed{S = \left[1 ; \frac{4}{3}\right] \cup [4 ; +\infty[}$$

Exercice 2

1°) Développement

a) $a^2+b^2+c^2+2ab+2bc+2ac$

b) $8x^3-12x^2+6x-1$

2°) Factorisation

$C(x) = (3x-4)(2x+1+3x+4)$

$= (3x-4)(5x+5)$

$= 5(3x-4)(x+1)$

$D(x) = (x-2)(5x+3)$ par les racines

Exercice 3

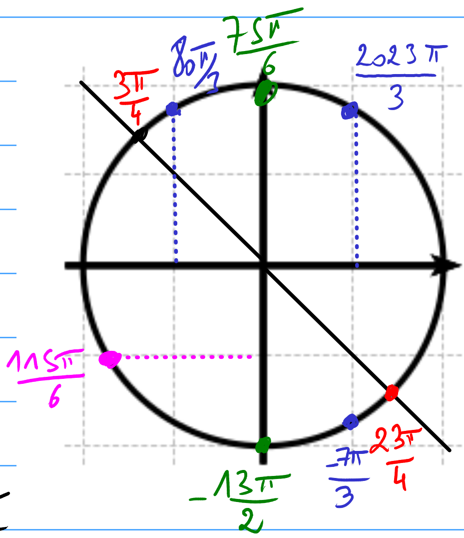
$\frac{23\pi}{4} = \frac{24\pi}{4} - \frac{\pi}{4} = 3\times\frac{8\pi}{4} - \frac{\pi}{4} \equiv -\frac{\pi}{4}[2\pi]$

$\frac{2023\pi}{3} \equiv \frac{\pi}{3}[2\pi]$ car 2022 est dans la table de 6 !

$\frac{75\pi}{6} = \frac{25\pi}{2} \equiv \frac{\pi}{2}[2\pi]$

$\frac{115\pi}{6} = \frac{120\pi}{6} - \frac{5\pi}{6}$

$\frac{80\pi}{3} = \frac{78\pi}{3} + \frac{2\pi}{3}$



Exercice 4 :

$A = \sin x - \cos x + 2\sin x = \boxed{3\sin x - \cos x}$

$B = \sin\frac{\pi}{8} - \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} - \sin\frac{7\pi}{8} = \boxed{0}$

$\sin(\pi - x) = \sin x$

Exercice 5 :

1°) $(\vec{AE}, \vec{AC}) = \frac{\pi}{4} + \frac{\pi}{3} \equiv \frac{7\pi}{12} [2\pi]$ $\boxed{MP : \frac{7\pi}{12}}$

2°) $(\vec{AD}, \vec{EA}) \equiv (\vec{AD}, \vec{AE}) + \pi [2\pi] \equiv \pi - \frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{3} \equiv \frac{\pi}{6} [2\pi]$ $\boxed{MP : \frac{\pi}{6}}$

3°) $(\vec{AE}; \vec{BC}) \equiv (\vec{AE}, \vec{EB}) + (\vec{EB}, \vec{BC}) \equiv (\vec{EA}, \vec{EB}) + \cancel{\pi} + (\vec{BE}, \vec{BC}) + \cancel{\pi} [2\pi]$

$\equiv -\frac{\pi}{2} - \frac{\pi}{4} - \frac{\pi}{3} \equiv -\frac{13\pi}{12} \equiv \frac{11\pi}{12} [2\pi]$ $\boxed{MP = \frac{11\pi}{12}}$

4°) $(\vec{BE}, \vec{CD}) \equiv (\vec{BE}, \vec{BC}) + (\vec{BC}, \vec{CD}) \equiv -\frac{\pi}{4} - \frac{\pi}{3} + (\vec{CB}, \vec{CD}) + \pi [2\pi]$

$\equiv -\frac{7\pi}{12} + \pi - \frac{\pi}{3} - \frac{\pi}{4} \equiv -\frac{2\pi}{12} \equiv -\frac{\pi}{6} [2\pi]$ $\boxed{MP : -\frac{\pi}{6}}$

Exercice 6 :

1°) $\cos x = -\frac{1}{2} = \cos\frac{2\pi}{3} \iff x = \pm\frac{2\pi}{3}$ $\boxed{S = \left\{-\frac{2\pi}{3} ; \frac{2\pi}{3}\right\}}$

2°) $\sin x = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4} \iff x = \frac{\pi}{4}$ ou $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\boxed{S = \left\{\frac{\pi}{4} ; \frac{3\pi}{4}\right\}}$

3°) $\cos 3x = \frac{1}{2} = \cos\frac{\pi}{3} \iff 3x = \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$ ou $3x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$\iff x = \frac{\pi}{9} + k\frac{2\pi}{3}, k \in \mathbb{Z}$ ou $x = -\frac{\pi}{9} + k\frac{2\pi}{3}, k \in \mathbb{Z}$

$\boxed{S = \left\{\frac{\pi}{9} ; \frac{7\pi}{9} ; -\frac{5\pi}{9} ; -\frac{7\pi}{9} ; \frac{-\pi}{9} ; \frac{5\pi}{9}\right\}}$

4°) $\cos(4x) = \sin\left(2x + \frac{\pi}{4}\right) \iff \cos(4x) = \cos\left(\frac{\pi}{2} - 2x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4} - 2x\right)$

$\iff 4x = \frac{\pi}{4} - 2x + k \times 2\pi, k \in \mathbb{Z}$, ou $4x = -\frac{\pi}{4} + 2x + k2\pi$, $k \in \mathbb{Z}$

$\iff 6x = \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$ ou $2x = -\frac{\pi}{4} + k \times 2\pi, k \in \mathbb{Z}$

$\Leftrightarrow \quad x = \frac{\pi}{24} + k \times \frac{\pi}{3}$ ou $x = -\frac{\pi}{8} + k\pi$
$k \in \mathbb{Z}$ $k \in \mathbb{Z}$

$$\boxed{S = \left\{ \frac{\pi}{24} ; -\frac{7\pi}{24} ; -\frac{15\pi}{24} ; -\frac{23\pi}{24} ; \frac{9\pi}{24} ; \frac{17\pi}{24} ; -\frac{\pi}{8} ; \frac{7\pi}{8} \right\}}$$

5°) $X = \cos x$ $\quad 2X^2 + X - 1 = 0$ a pour solutions -1 et $\frac{1}{2}$

donc $\cos x = -1 \Leftrightarrow x = \pi + k \times 2\pi, k \in \mathbb{Z}$

ou $\cos x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{3}$ ou $-\frac{\pi}{3} + k \times 2\pi, k \in \mathbb{Z}$

$$\boxed{S = \left\{ -\frac{\pi}{3} ; \frac{\pi}{3} ; \pi \right\}}$$

Exercice 7 :

1°) $\sin\left(\frac{\pi}{3} + x\right) = \sin\frac{\pi}{3}\cos x + \sin x \cos\frac{\pi}{3}$

$\sin\left(\frac{\pi}{3} - x\right) = \sin\frac{\pi}{3}\cos x - \sin x \cos\frac{\pi}{3}$

$\sin\left(\frac{\pi}{3} + x\right) - \sin\left(\frac{\pi}{3} - x\right) = 2 \underbrace{\cos\frac{\pi}{3}}_{1/2} \sin x = \sin x$

2°) a) $A = \sin(a+b)\sin(a-b)$

$A = (\sin a \cos b + \sin b \cos a)(\sin a \cos b - \sin b \cos a)$

$A = \sin^2 a \cos^2 b - \sin^2 b \cos^2 a$

$$A = \sin^2 a(1-\sin^2 b) - \sin^2 b(1-\sin^2 a)$$

$$A = \sin^2 a \quad \cancel{-\sin^2 a \sin^2 b} - \sin^2 b + \cancel{\sin^2 b \sin^2 a}$$

On a bien $\boxed{\sin(a+b)\sin(a-b) = \sin^2 a - \sin^2 b}$

b) Si $a = \frac{\pi}{3}$ et $b = \frac{\pi}{4}$, $a+b = \frac{7\pi}{12}$

$a - b = \frac{\pi}{12}$

$$\sin(a+b)\sin(a-b) = \sin^2 a - \sin^2 b.$$

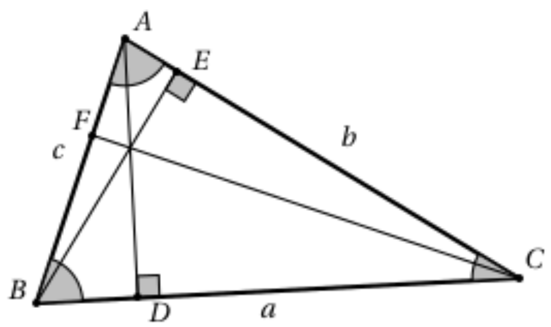
$$\sin\left(\frac{7\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) = \sin^2\frac{\pi}{3} - \sin^2\frac{\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2$$

$$= \frac{3}{4} - \frac{2}{4}$$

$$\boxed{\sin\left(\frac{7\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) = \frac{1}{4}}$$

Exercice 8

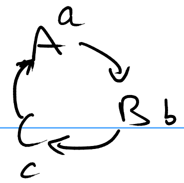


1°) FORMULE 1 :

a) dans BAD, rect. en D, $\sin\widehat{B} = \frac{AD}{BA} \iff \underline{AD = c \times \sin\widehat{B}}$

b) $\boxed{S = \frac{BC \times AD}{2} = \frac{1}{2} a \times c \times \sin\widehat{B}}$

c) Par permutation circulaire :



$$S=\frac{1}{2}ac\sin\hat{B} = \frac{1}{2}\underbrace{ba\sin\hat{C}}_{b\times BE} = \frac{1}{2}c\underbrace{b\sin\hat{A}}_{CF}.$$

2°) Loi des sinus

$$S=\frac{1}{2}ac\sin\hat{B}=\frac{1}{2}ba\sin\hat{C}=\frac{1}{2}cb\sin\hat{A}$$

×2

$$2S = ac\sin\hat{B} = ba\sin\hat{C} = cb\sin\hat{A}$$

÷abc

$$\frac{2S}{abc} = \frac{\cancel{a}\cancel{c}\sin\hat{B}}{\cancel{a}b\cancel{c}} = \frac{\cancel{b}\cancel{a}\sin\hat{C}}{\cancel{a}\cancel{b}c} = \frac{\cancel{c}\cancel{b}\sin\hat{A}}{a\cancel{b}\cancel{c}}$$

on inverse

$$\boxed{\frac{abc}{2S}=\frac{b}{\sin\hat{B}}=\frac{c}{\sin\hat{C}}=\frac{a}{\sin\hat{A}}}$$

3°) Formule 2 :

a) $\hat{A}+\hat{B}+\hat{C}=\pi$ dans le triangle ABC

Donc $\underline{\hat{A} = \pi-(\hat{B}+\hat{C})}$

b) $\sin\hat{A} = \sin(\pi-(\hat{B}+\hat{C}))$ or $\sin(\pi-x)=\sin x$

donc $\underline{\sin\hat{A}=\sin(\hat{B}+\hat{C})}$

c) on a, d'après la formule 1, $S=\frac{1}{2}ac\sin\hat{B}$

De plus d'après la loi des sinus, $\frac{a}{\sin\hat{A}}=\frac{c}{\sin\hat{C}}$

$$\Leftrightarrow c=\frac{a\sin\hat{C}}{\sin\hat{A}}$$

Ainsi $S = \frac{1}{2} a \times \left(\frac{a \sin \widehat{C}}{\sin \widehat{A}} \right) \times \sin \widehat{B}$

$\iff S = \frac{a^2}{2} \times \frac{\sin \widehat{B} \times \sin \widehat{C}}{\sin \widehat{A}}$

or d'après 3-b, $\sin \widehat{A} = \sin(\widehat{B} + \widehat{C})$

Ainsi $\boxed{S = \frac{a^2}{2} \times \frac{\sin \widehat{B} \times \sin \widehat{C}}{\sin(\widehat{B}+\widehat{C})}}$

4°) FORMULE DE HERON

a) D'après la formule d'Al Kashi,

$$a^2 = b^2 + c^2 - 2bc \cos \widehat{A}$$

$\iff 2bc \cos \widehat{A} = b^2 + c^2 - a^2$

$$\iff \cos \widehat{A} = \frac{b^2 + c^2 - a^2}{2bc}$$

Par ailleurs $S = \frac{1}{2} bc \sin \widehat{A} \iff \frac{2S}{bc} = \sin \widehat{A}$

b) on a $\cos^2 \widehat{A} + \sin^2 \widehat{A} = 1$

$$\iff \left(\frac{b^2 + c^2 - a^2}{2bc} \right)^2 + \left(\frac{2S}{bc} \right)^2 = 1$$

$$\iff \left(\frac{b^2 + c^2 - a^2}{2bc} \right)^2 + \frac{4S^2}{b^2c^2} = 1$$

c) Ainsi $\dfrac{(b^2+c^2-a^2)^2}{4b^2c^2} + \dfrac{4\times 4S^2}{4\times b^2c^2} = 1$

$\iff (b^2+c^2-a^2)^2 + 16S^2 = 4b^2c^2$

$\iff \boxed{16S^2 = 4b^2c^2 - (b^2+c^2-a^2)^2}$

d) On va factoriser l'expression de droite $A^2-B^2 = (A-B)(A+B)$

$$16S^2 = (2bc)^2 - (b^2+c^2-a^2)^2$$

$$= (\underbrace{2bc - b^2 - c^2}_{-(b-c)^2} + a^2)(\underbrace{2bc + b^2 + c^2}_{(b+c)^2} - a^2)$$

$$= (a^2 - (b-c)^2)((b+c)^2 - a^2)$$

$$= (a-b+c)(a+b-c)(b+c-a)(b+c+a)$$

Donc $\boxed{16S^2 = (a+b+c)(a+b-c)(a+c-b)(b+c-a)}$

e) On pose $p = \dfrac{a+b+c}{2}$ alors $a+b-c = 2p-2c$ par exemple

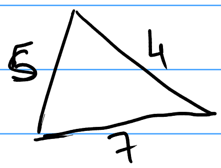
Alors $16S^2 = 2p \times 2(p-c) \times 2(p-b) \times 2(p-a)$

$\iff \cancel{16}S^2 = \cancel{16}\, p(p-a)(p-b)(p-c)$

$\iff \boxed{S = \sqrt{p(p-a)(p-b)(p-c)}}$

5°) Application numérique

a)

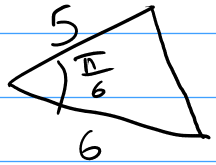


$$S=\sqrt{p(p-a)(p-b)(p-c)}$$

$$p=\frac{7+5+4}{2}=8$$

$$S=\sqrt{8\times 1\times 3\times 4}=\boxed{4\sqrt{6}\ cm^2}$$

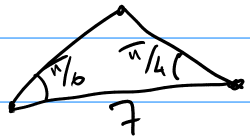
b)



$$S=\frac{1}{2}ac\sin\hat{B}=\frac{1}{\cancel{2}}\,\cancel{6}^{3}\times 5\times\sin\frac{\pi}{6}$$

$$=\frac{15}{2}=\boxed{7{,}5\ cm^2}$$

c)



$$S=\frac{a^2}{2}\,\frac{\sin\hat{B}\,\sin\hat{C}}{\sin(\hat{B}+\hat{C})}=\frac{49}{2}\times\frac{\sin\frac{\pi}{6}\sin\frac{\pi}{4}}{\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)}$$

Or $\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\frac{\pi}{6}$

$$=\frac{1}{2}\,\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{2}+\sqrt{6}}{4}$$

Ainsi $S=\dfrac{49}{2}\times\dfrac{\frac{1}{\cancel{2}}\times\frac{\cancel{\sqrt{2}}\,1}{\cancel{2}}}{\frac{1\cancel{\sqrt{2}}+\cancel{\sqrt{6}}\sqrt{3}}{\cancel{4}}}=\dfrac{49}{2}\times\dfrac{1}{\sqrt{3}+1}=\dfrac{49}{2}\times\dfrac{\sqrt{3}-1}{3-1}$

$$=\boxed{\frac{49(\sqrt{3}-1)}{4}\simeq 8{,}97\ cm^2}$$